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Vector Algebra

TOPIC 1 Algebra of Vectors, Section Formula, Linear Dependence & Independence of Vectors, Position Vector of a Point, Modulus of a Vector, Collinearity of Three points, Coplanarity of Three Vectors & Four Points, Vector Inequality

1. Let $a, b, c \in \mathbf{R}$ be such that $a^2+b^2+c^2=1$. If $a\cos\theta - b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right)$, where $\theta=\frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is : **[Sep. 03, 2020 (II)]**

   (a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{9}$ (d) 0

2. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda : 1\ (\lambda > 0)$. If O is the region and $\overrightarrow{OB}\cdot\overrightarrow{OP} - 3|\overrightarrow{OA}\times\overrightarrow{OP}|^2 = 6$, then λ is equal to ___________. **[NA Sep. 02, 2020 (II)]**

3. If the vectors, $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$, $\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and $\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$ $(a \in \mathrm{R})$ are coplanar and $3(\vec{p}.\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$, then the value of λ is _____. **[NA Jan. 9, 2020 (I)]**

4. Let $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ has the magnitude 12 then one such vector is : **[April 12, 2019 (I)]**

   (a) $4\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$ (b) $4\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

   (c) $4\left(2\hat{i}+2\hat{j}-\hat{k}\right)$ (d) $4\left(-2\hat{i}-2\hat{j}+\hat{k}\right)$

5. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}, \hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to : **[April 12, 2019 (I)]**

   (a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$

6. Let $\alpha \in \mathrm{R}$ and the three vectors $\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c}=\alpha\hat{i}-2\hat{j}+3\hat{k}$. Then the set $S=(\alpha : \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar) **[April 12, 2019 (II)]**

   (a) is singleton
   (b) is empty
   (c) contains exactly two positive numbers
   (d) contains exactly two numbers only one of which is positive

7. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of , is: **[April 09, 2019 (II)]**

   (a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{5\pi}{12}$ (d) $\frac{2\pi}{3}$

8. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$, are co-planar, is : **[Jan. 12, 2019 (I)]**

   (a) –1 (b) 0 (c) 1 (d) 2

9. Let $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}, \vec{b}=\hat{i}+\lambda\hat{j}+4\hat{k}$ and $\vec{c}=2\hat{i}+4\hat{j}+\left(\lambda^2-1\right)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a}\times\vec{c}$ is : **[Jan. 11, 2019 (I)]**

   (a) $-10\hat{i}-5\hat{j}$ (b) $-14\hat{i}-5\hat{j}$

   (c) $-14\hat{i}+5\hat{j}$ (d) $-10\hat{i}+5\hat{j}$

10. Let $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is : **[Jan. 11, 2019 (II)]**
(a) 4 (b) 3 (c) 2 (d) 1

11. Let $\vec{\alpha}=(\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is: **[Jan. 10, 2019 (II)]**
(a) -4 (b) -3 (c) 4 (d) 3

12. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}-24$, then $|\vec{u}|^2$ is equal to : **[2018]**
(a) 315 (b) 256 (c) 84 (d) 336

13. Let ABC be a triangle whose circumcentre is at P. If the position vectors A, B, C and P are $\vec{a},\vec{b},\vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is : **[Online April 10, 2016]**
(a) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$ (b) $\vec{a}+\vec{b}+\vec{c}$
(c) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$ (d) $\vec{0}$

14. If the vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is **[2013]**
(a) $\sqrt{18}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{45}$

15. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then the value of α for which the vectors $\vec{u}=(\alpha-2)\vec{a}+\vec{b}$ and $\vec{v}=(2+3\alpha)\vec{a}-3\vec{b}$ are collinear is : **[Online April 23, 2013]**
(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $-\frac{3}{2}$ (d) $-\frac{2}{3}$

16. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c}=r\hat{i}+\hat{j}+(2r-1)\hat{k}$ are three vectors such that $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$, then r is equal to **[Online May 19, 2012]**
(a) 1 (b) -1 (c) 0 (d) 2

17. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+3\vec{b}+6\vec{c}$ is : **[2011RS]**
(a) $\vec{a}$ (b) $\vec{c}$ (c) $\vec{0}$ (d) $\vec{a}+\vec{c}$

18. If the $p\hat{i}+\hat{j}+\hat{k}, \hat{i}+q\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+r\hat{k}\ (p\neq q\neq r\neq 1)$ vector are coplanar, then the value of $pqr-(p+q+r)$ is **[2011RS]**
(a) 2 (b) 0 (c) -1 (d) -2

19. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β? **[2008]**
(a) $\alpha=2, \beta=2$ (b) $\alpha=1, \beta=2$
(c) $\alpha=2, \beta=1$ (d) $\alpha=1, \beta=1$

20. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overline{AB},\overline{BC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overline{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is **[2006]**
(a) $\frac{AB^2+AC^2}{(AB)^2(AC)^2}$ (b) $\frac{(AB)(AC)}{AB+AC}$
(c) $\frac{1}{AB}+\frac{1}{AC}$ (d) $\frac{1}{AD}$

21. If C is the mid point of AB and P is any point outside AB, then **[2005]**
(a) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$
(b) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$
(c) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$
(d) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{0}$

22. Let a, b and c be distinct non- negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is **[2005]**
(a) the Geometric Mean of a and b
(b) the Arithmetic Mean of a and b
(c) equal to zero
(d) the Harmonic Mean of a and b

23. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non coplanar for **[2004]**
(a) no value of λ
(b) all except one value of λ
(c) all except two values of λ
(d) all values of λ

24. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals **[2004]**
(a) 0 (b) $\lambda\vec{b}$ (c) $\lambda\vec{c}$ (d) $\lambda\vec{a}$

25. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a **[2003]**
(a) parallelogram but not a rhombus
(b) square
(c) rhombus
(d) rectangle.

26. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non- coplanar, then the product abc equals **[2003]**
(a) 0 (b) 2 (c) –1 (d) 1

27. The vectors $\overrightarrow{AB} = 3\hat{i}+4\hat{k}$ & $\overrightarrow{AC} = 5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is **[2003]**
(a) $\sqrt{288}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$

TOPIC 2 Scalar or Dot Product of two Vectors, Projection of a Vector Along any other Vector, Component of a Vector

28. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is ______. **[NA Sep. 06, 2020 (I)]**

29. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}| = |\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is ______. **[NA Sep. 06, 2020 (II)]**

30. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a}+\vec{b}-\vec{c}|$ is ______. **[NA Sep. 05, 2020 (II)]**

31. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2 = 8$. Then $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$ is equal to ______. **[NA Sep. 02, 2020 (I)]**

32. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is ______. **[NA Jan. 9, 2020 (I)]**

33. Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u} = \hat{i}+\hat{j}+\lambda\hat{k}, \vec{v} = \hat{i}+\hat{j}+3\hat{k}$ and $\vec{w} = 2\hat{i}+\hat{j}+\hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be: **[Jan. 8, 2020 (I)]**
(a) $\frac{7}{6\sqrt{6}}$ (b) $\frac{7}{6\sqrt{3}}$ (c) $\frac{5}{7}$ (d) $\frac{5}{3\sqrt{3}}$

34. A vector $\vec{a} = \alpha\hat{i}+2\hat{j}+\beta\hat{k}$ $(\alpha, \beta \in \boldsymbol{R})$ lies in the plane of the vectors, $\vec{b} = \hat{i}+\hat{j}$ and $\vec{c} = \hat{i}-\hat{j}+4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then: **[Jan. 7, 2020 (I)]**
(a) $\vec{a}\cdot\hat{i}+3 = 0$ (b) $\vec{a}\cdot\hat{i}+1 = 0$
(c) $\vec{a}\cdot\hat{k}+2 = 0$ (d) $\vec{a}\cdot\hat{k}+4 = 0$

35. Let $\vec{a} = 2\hat{i}+\lambda_1\hat{j}+3\hat{k}, \vec{b} = 4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c} = 3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$ Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is: **[Jan. 10, 2019 (I)]**
(a) (1, 3, 1) (b) $\left(-\frac{1}{2}, 4, 0\right)$
(c) $\left(\frac{1}{2}, 4, -2\right)$ (d) (1, 5, 1)

36. Let $\vec{a} = \hat{i}+\hat{j}+\sqrt{2}\hat{k}, \vec{b} = b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$.
If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to: **[Jan. 09, 2019 (II)]**
(a) $\sqrt{32}$ (b) 6 (c) $\sqrt{22}$ (d) 4

37. In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}, -\hat{i}+3\hat{j}+p\hat{k}$ and $5\hat{i}+q\hat{j}-4\hat{k}$, then the point (p, q) lies on a line : **[Online April 9, 2016]**
(a) making an obtuse angle with the positive direction of x–axis
(b) parallel to x–axis
(c) parallel to y–axis
(d) making an acute angle with the positive direction of x–axis

38. In a parallelogram ABD, $|\overrightarrow{AB}|=a, |\overrightarrow{AD}|=b$ and $|\overrightarrow{AC}|=c$, then $\overrightarrow{DA}.\overrightarrow{AB}$ has the value : **[Online April 11, 2015]**
(a) $\frac{1}{2}(a^2+b^2+c^2)$ (b) $\frac{1}{2}(a^2-b^2+c^2)$
(c) $\frac{1}{2}(a^2+b^2-c^2)$ (d) $\frac{1}{3}(b^2+c^2-a^2)$

39. If $\hat{x}, \hat{y}$ and $\hat{z}$ are three unit vectors in three-dimensional space, then the minimum value of
$|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2$ **[Online April 12, 2014]**
(a) $\frac{3}{2}$ (b) 3 (c) $3\sqrt{3}$ (d) 6

40. If $|\vec{a}|=2, |\vec{b}|=3$ and $|2\vec{a}-\vec{b}|=5$, then $|2\vec{a}+\vec{b}|$ equals: **[Online April 9, 2014]**
(a) 17 (b) 7 (c) 5 (d) 1

41. If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors satisfying $\hat{a}-\sqrt{3}\hat{b}+\hat{c}=\vec{0}$, then the angle between the vectors $\hat{a}$ and $\hat{c}$ is : **[Online April 22, 2013]**
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

42. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector of the type $\vec{b}+\lambda\vec{c}$ for some scalar λ, whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is : **[Online April 9, 2013]**
(a) $2\hat{i}+\hat{j}+5\hat{k}$ (b) $2\hat{i}+3\hat{j}-3\hat{k}$
(c) $2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+3\hat{j}+3\hat{k}$

43. Let *ABCD* be a parallelogram such that $\overrightarrow{AB}=\vec{q}, \overrightarrow{AD}=\vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincide with the altitude directed from the vertex B to the side *AD*, then $\vec{r}$ is given by : **[2012]**
(a) $\vec{r}=3\vec{q}-\frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (b) $\vec{r}=-\vec{q}+\frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$
(c) $\vec{r}=\vec{q}-\frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (d) $\vec{r}=-3\vec{q}-\frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

44. Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is : **[2012]**
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

45. If $a+b+c=0$, $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is **[Online May 19, 2012]**
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

46. A unit vector which is perpendicular to the vector $2\hat{i}-\hat{j}+2\hat{k}$ and is coplanar with the vectors $\hat{i}+\hat{j}-\hat{k}$ and $2\hat{i}+2\hat{j}-\hat{k}$ is **[Online May 12, 2012]**
(a) $\frac{2\hat{j}+\hat{k}}{\sqrt{5}}$ (b) $\frac{3\hat{i}+2\hat{j}-2\hat{k}}{\sqrt{17}}$
(c) $\frac{3\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{17}}$ (d) $\frac{2\hat{i}+2\hat{j}-2\hat{k}}{3}$

47. *ABCD* is parallelogram. The position vectors of *A* and *C* are respectively, $3\hat{i}+3\hat{j}+5\hat{k}$ and $\hat{i}-5\hat{j}-5\hat{k}$. If *M* is the midpoint of the diagonal *DB*, then the magnitude of the projection of $\overrightarrow{OM}$ on $\overrightarrow{OC}$, where *O* is the origin, is **[Online May 7, 2012]**
(a) $7\sqrt{51}$ (b) $\frac{7}{\sqrt{50}}$ (c) $7\sqrt{50}$ (d) $\frac{7}{\sqrt{51}}$

48. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then $(\lambda,\mu)=$ **[2010]**
(a) (2, –3) (b) (–2, 3)
(c) (3, –2) (d) (–3, 2)

49. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is **[2008]**
(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

50. The values of a, for which points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right angled triangle with $C=\frac{\pi}{2}$ are **[2006]**

(a) 2 and 1 (b) – 2 and – 1
(c) – 2 and 1 (d) 2 and – 1

51. Let $\bar{u},\bar{v},\bar{w}$ be such that $|\bar{u}|=1, |\bar{v}|=2, |\bar{w}|=3$. If the projection $\bar{v}$ along $\bar{u}$ is equal to that of $\bar{w}$ along $\bar{u}$ and $\bar{v}$, $\bar{w}$ are perpendicular to each other then $|\bar{u}-\bar{v}+\bar{w}|$ equals **[2004]**

(a) 14 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 2

52. $\vec{a},\vec{b},\vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1$, $|\vec{b}|=2, |\vec{c}|=3$, then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to **[2003]**

(a) 1 (b) 0 (c) –7 (d) 7

53. If $|\vec{a}|=5$, $|\vec{b}|=4$, $|\vec{c}|=3$ thus what will be the value of $|\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$ **[2002]**

(a) 25 (b) 50 (c) –25 (d) –50

54. If sdaa $\vec{a},\vec{b},\vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is **[2002]**

(a) 60° (b) 30° (c) 45° (d) 90°

TOPIC 3 Vector or Cross Product of two vectors, Area of a Parallelogram & Triangle, Scalar & Vector Tripple Product

55. If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a}=\hat{i}+\hat{j}+n\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c}=\hat{i}+n\hat{j}+3\hat{k}\,(n\geq 0)$, is 158 cu.units, then: **[Sep. 05, 2020 (I)]**

(a) $\vec{a}\cdot\vec{c}=17$ (b) $\vec{b}\cdot\vec{c}=10$
(c) $n=7$ (d) $n=9$

56. Let x_0 be the point of local maxima of $f(x)=\vec{a}\cdot(\vec{b}\times\vec{c})$, where $\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x=x_0$ is: **[Sep. 04, 2020 (I)]**

(a) – 4 (b) – 30 (c) 14 (d) – 22

57. If $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$, then the value of $|\hat{i}\times(\vec{a}\times\hat{i})|^2+|\hat{j}\times(\vec{a}\times\hat{j})|^2+|\hat{k}\times(\vec{a}\times\hat{k})|^2$ is equal to ________________. **[NA Sep. 04, 2020 (II)]**

58. Let $\vec{b}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{3}, |\vec{b}|=5$, $\vec{b}\cdot\vec{c}=10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{b}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to _____. **[NA Jan. 9, 2020 (II)]**

59. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}\cdot\vec{a}=0$, then $\vec{c}\cdot\vec{b}$ is equal to: **[Jan. 8, 2020 (II)]**

(a) $-\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) –1

60. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. if $\lambda=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda,\vec{d})$ is equal to: **[Jan. 7, 2020 (II)]**

(a) $\left(\frac{3}{2},3\vec{a}\times\vec{c}\right)$ (b) $\left(-\frac{3}{2},3\vec{c}\times\vec{b}\right)$
(c) $\left(\frac{3}{2},3\vec{b}\times\vec{c}\right)$ (d) $\left(-\frac{3}{2},3\vec{a}\times\vec{b}\right)$

61. Let $\vec{\alpha}=3\hat{i}+\hat{j}$ and $\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$. If $\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1\times\vec{\beta}_2$ is equal to: **[April 09, 2019 (I)]**

(a) $-3\hat{i}+9\hat{j}+5\hat{k}$ (b) $3\hat{i}-9\hat{j}-5\hat{k}$
(c) $\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$ (d) $\frac{1}{2}(3\hat{i}-9\hat{j}+5\hat{k})$

62. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is: **[April 08, 2019 (I)]**

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{6}$ (c) $3\sqrt{6}$ (d) $\sqrt{\frac{3}{2}}$

63. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if : **[April 08, 2019 (II)]**

(a) $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$ (b) $r \ge 5\sqrt{\frac{3}{2}}$

(c) $0 < r \le \sqrt{\frac{3}{2}}$ (d) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$

64. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, then $|\alpha - \beta|$ is equal to : **[Jan. 12, 2019 (II)]**

(a) 30° (b) 90° (c) 60° (d) 45°

65. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to: **[Jan 09, 2019]**

(a) $\frac{19}{2}$ (b) 9 (c) 8 (d) $\frac{17}{2}$

66. If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i} + 7\hat{j} + 8\hat{k}$, $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is **[Online April 15, 2018]**

(a) $\frac{1}{2}(4\hat{i} + 8\hat{j} + 11\hat{k})$ (b) $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

(c) $\frac{1}{4}(8\hat{i} + 14\hat{j} + 9\hat{k})$ (d) $\frac{1}{3}(6\hat{i} + 11\hat{j} + 15\hat{k})$

67. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$. Then $|\vec{b}|$ equals? **[Online April 16, 2018]**

(a) $\sqrt{\frac{11}{3}}$ (b) $\frac{\sqrt{11}}{3}$ (c) $\frac{11}{\sqrt{3}}$ (d) $\frac{11}{3}$

68. If $\vec{a}, \vec{b}$, and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$, then $|\vec{a} \times \vec{c}|$ is equal to **[Online April 15, 2018]**

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{15}}{4}$ (c) $\frac{15}{16}$ (d) $\frac{\sqrt{15}}{16}$

69. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a}.\vec{c}$ is equal to : **[2017]**

(a) $\frac{1}{8}$ (b) $\frac{25}{8}$ (c) 2 (d) 5

70. If the vector $\vec{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector $\vec{b_1}$, parallel to $\vec{a} = \hat{i} + \hat{j}$ and a vector $\vec{b_2}$, perpendicular to $\vec{a}$, then $\vec{b_1} \times \vec{b_2}$ is equal to : **[Online April 9, 2017]**

(a) $-3\hat{i} + 3\hat{j} - 9\hat{k}$ (b) $6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

(c) $-6\hat{i} + 6\hat{j} - \frac{9}{2}\hat{k}$ (d) $3\hat{i} - 3\hat{j} + 9\hat{k}$

71. The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is : **[Online April 8, 2017]**

(a) 26 (b) 65 (c) 20 (d) 52

72. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[2016]**

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{2}$

73. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is : **[2015]**

(a) $\frac{2}{3}$ (b) $\frac{-2\sqrt{3}}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-\sqrt{2}}{3}$

74. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$. If $\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$, then $2|\vec{c}|$ is equal to : **[Online April 10, 2015]**

(a) $\sqrt{55}$ (b) $\sqrt{37}$ (c) $\sqrt{51}$ (d) $\sqrt{43}$

75. If $[\vec{a} \times \vec{b} \;\; \vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a}] = \lambda[\vec{a}\vec{b}\vec{c}]^2$ then λ is equal to **[2014]**

(a) 0 (b) 1 (c) 2 (d) 3

76. If $\vec{x} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} - 12\hat{k}$, then the magnitude of the projection of $\vec{x} \times \vec{y}$ on $\vec{z}$ is: **[Online April 19, 2014]**

(a) 12 (b) 15 (c) 14 (d) 13

77. If $|\vec{c}|^2 = 60$ and $\vec{c} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = \vec{0}$, then a value of $\vec{c}.(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is: **[Online April 11, 2014]**
(a) $4\sqrt{2}$ (b) 12 (c) 24 (d) $12\sqrt{2}$

78. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \bullet \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ equals: **[Online April 25, 2013]**
(a) $\frac{1}{2}$ (b) $\frac{3\sqrt{3}}{2}$ (c) 3 (d) $\frac{3}{2}$

79. The vector $(\hat{i} \times \vec{a}.\vec{b})\hat{i} + (\hat{j} \times \vec{a}.\vec{b})\hat{j} + (\hat{k} \times \vec{a}.\vec{b})\hat{k}$ is equal to : **[Online April 9, 2013]**
(a) $\vec{b} \times \vec{a}$ (b) $\vec{a}$ (c) $\vec{a} \times \vec{b}$ (d) $\vec{b}$

80. **Statement 1:** The vectors $\vec{a}, \vec{b}$ and $\vec{c}$ lie in the same plane if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

Statement 2: The vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if $\vec{u}.\vec{v} = 0$ where $\vec{u} \times \vec{v}$ is a vector perpendicular to the plane of $\vec{u}$ and $\vec{v}$. **[Online May 26, 2012]**
(a) Statement 1 is false, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is false.
(d) Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1.

81. If $\vec{u} = \hat{j} + 4\hat{k}$, $\vec{v} = \hat{i} + 3\hat{k}$ and $\vec{w} = \cos\theta\hat{i} + \sin\theta\hat{j}$ are vectors in 3-dimensional space, then the maximum possible value of $|\vec{u} \times \vec{v}.\vec{w}|$ is **[Online May 12, 2012]**
(a) $\sqrt{3}$ (b) 5 (c) $\sqrt{14}$ (d) 7

82. **Statement 1:** If the points (1, 2, 2), (2, 1, 2) and (2, 2, z) and (1, 1, 1) are coplanar, then $z = 2$.
Statement 2: If the 4 points P, Q, R and S are coplanar, then the volume of the tetrahedron $PQRS$ is 0. **[Online May 12, 2012]**
(a) Statement 1 is false,, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

83. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + (2\lambda - 1)\hat{k}$ are coplanar vectors, then λ is equal to **[Online May 7, 2012]**
(a) 0 (b) – 1 (c) 2 (d) 1

84. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a}.\vec{d} = 0$. Then the vector $\vec{d}$ is equal to **[2011]**
(a) $\vec{c} + \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (b) $\vec{b} + \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$
(c) $\vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (d) $\vec{b} - \left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

85. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b})[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is **[2011]**
(a) –3 (b) 5 (c) 3 (d) –5

86. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is **[2010]**
(a) $2\hat{i} - \hat{j} + 2\hat{k}$ (b) $\hat{i} - \hat{j} - 2\hat{k}$
(c) $\hat{i} + \hat{j} - 2\hat{k}$ (d) $-\hat{i} + \hat{j} - 2\hat{k}$

87. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality
$[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$
holds for : **[2009]**
(a) exactly two values of (p, q)
(b) more than two but not all values of (p, q)
(c) all values of (p, q)
(d) exactly one value of (p, q)

88. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals **[2007]**
(a) – 4 (b) – 2 (c) 0 (d) 1.

89. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for **[2007]**
(a) no value of θ
(b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ

90. If $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}.\vec{b} \neq 0$, $\vec{b}.\vec{c} \neq 0$ then $\vec{a}$ and $\vec{c}$ are **[2006]**
(a) inclined at an angle of $\frac{\pi}{3}$ between them
(b) inclined at an angle of $\frac{\pi}{6}$ between them
(c) perpendicular
(d) parallel

91. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on **[2005]**

(a) only y (b) only x
(c) both x and y (d) neither x nor y

92. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number then **[2005]**

$[\lambda(\vec{a} + \vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}] = [\vec{a}\quad \vec{b} + \vec{c}\quad \vec{b}]$ for

(a) exactly one value of λ
(b) no value of λ
(c) exactly three values of λ
(d) exactly two values of λ

93. For any vector $\vec{a}$, the value of

$(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to **[2005]**

(a) $3\vec{a}^2$ (b) $\vec{a}^2$ (c) $2\vec{a}^2$ (d) $4\vec{a}^2$

94. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be non-zero vectors such that $(\bar{a} \times \bar{b}) \times \bar{c} = \frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$. If θ is the acute angle between the vectors $\bar{b}$ and $\bar{c}$, then $\sin\theta$ equals **[2004]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

95. If $\vec{u}, \vec{v}$ *and* $\vec{w}$ are three non- coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w}).(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})$ equals **[2003]**

(a) $3\vec{u}.\vec{v} \times \vec{w}$ (b) 0
(c) $\vec{u}.(\vec{v} \times \vec{w})$ (d) $\vec{u}.\vec{w} \times \vec{v}$.

96. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1) B(2, 1, 3) and C(-1, 1, 2). Then the angle between the faces OAB and ABC will be **[2003]**

(a) 90^o (b) $\cos^{-1}\left(\frac{19}{35}\right)$

(c) $\cos^{-1}\left(\frac{17}{31}\right)$ (d) 30^o

97. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$, then $|\vec{w}.\hat{n}|$ is equal to **[2003]**

(a) 3 (b) 0 (c) 1 (d) 2.

98. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ then $\vec{a} + \vec{b} + \vec{c} =$ **[2002]**

(a) abc (b) –1 (c) 0 (d) 2

99. $\vec{a} = 3\hat{i} - 5\hat{j}$ and $\vec{b} = 6\hat{i} + 3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a} \times \vec{b}$ then $|\vec{a}| : |\vec{b}| : |\vec{c}|$ **[2002]**

(a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$ (b) $\sqrt{34} : \sqrt{45} : 39$
(c) $34 : 39 : 45$ (d) $39 : 35 : 34$

100. If the vectors $\vec{c}$, $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\hat{b} = \hat{j}$ are such that $\vec{a}$, $\vec{c}$ and $\vec{b}$ form a right handed system then $\vec{c}$ is: **[2002]**

(a) $z\hat{i} - x\hat{k}$ (b) $\vec{0}$
(c) $y\hat{j}$ (d) $-z\hat{i} + x\hat{k}$

101. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}] = 4$ then $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] =$ **[2002]**

(a) 16 (b) 64 (c) 4 (d) 8

102. If $|\vec{a}| = 4, |\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then $(\vec{a} \times \vec{b})^2$ is equal to **[2002]**

(a) 48 (b) 16
(c) $\vec{a}$ (d) None of these

TOPIC 4 Scalar Product of Four Vectors, Reciprocal System of Vector, Application of Vectors in Mechanics

103. A particle just clears a wall of height b at a distance a and strikes the ground at a distance *c* from the point of projection. The angle of projection is **[2007]**

(a) $\tan^{-1}\frac{bc}{a(c-a)}$ (b) $\tan^{-1}\frac{bc}{a}$

(c) $\tan^{-1}\frac{b}{ac}$ (d) 45°.

104. A body weighing 13 kg is suspended by two strings 5m and 12m long, their other ends being fastened to the extremities of a rod 13m long. If the rod be so held that the body hangs immediately below the middle point, then tensions in the strings are **[2007]**

(a) 5 kg and 12 kg (b) 5 kg and 13 kg
(c) 12 kg and 13 kg (d) 5 kg and 5 kg

105. The resultant of two forces Pn and $3n$ is a force of $7n$. If the direction of $3n$ force were reversed, the resultant would be $\sqrt{19}\,n$. The value of P is **[2007]**

(a) $3\,n$ (b) $4\,n$ (c) $5\,n$ (d) $6\,n$.

106. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4s prior to passing through P. If $g = 10m/s^2$, then the height above the point P from where the body began to fall is **[2006]**

(a) 720m (b) 900m (c) 320m (d) 680m

107. A particle has two velocities of equal magnitude inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is **[2006]**

(a) 90° (b) 120° (c) 45° (d) 60°

108. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is: **[2005]**

(a) $2:1$ (b) $3:\sqrt{2}$ (c) $3:2$ (d) $3:2\sqrt{2}$

109. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance **[2005]**

(a) $\frac{2H}{A-B}$ (b) $\frac{H}{A+B}$

(c) $\frac{H}{2(A+B)}$ (d) $\frac{H}{A-B}$

110. A particle is projected from a point O with velocity u at an angle of 60° with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by **[2005]**

(a) $\frac{u}{3}$ (b) $\frac{u}{2}$ (c) $\frac{2u}{3}$ (d) $\frac{u}{\sqrt{3}}$

111. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to **[2004]**

(a) 1 (b) $4u^2/g^2$

(c) $u^2/2g$ (d) u^2/g

112. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is **[2004]**

(a) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s (b) $\frac{1}{4}(\sqrt{3}-1)$ m/s

(c) $\frac{1}{4}$ m/s (d) $\frac{1}{8}$ m/s

113. A paticle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5km/hr. If $AB = 12$ km and $BC = 5$ km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively **[2004]**

(a) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h

(b) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h

(c) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h

(d) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h

114. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC are in equilibrium. Then $\vec{P}:\vec{Q}:\vec{R}$ is **[2004]**

(a) $\operatorname{cosec}\frac{A}{2}:\operatorname{cosec}\frac{B}{2}:\operatorname{cosec}\frac{C}{2}$

(b) $\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}$

(c) $\sec\frac{A}{2}:\sec\frac{B}{2}:\sec\frac{C}{2}$

(d) $\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$

115. In a right angle ΔABC, $\angle A = 90°$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is **[2004]**

(a) 9 (b) 4 (c) 5 (d) 3

116. With two forces acting at point, the maximum affect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are **[2004]**

(a) $\left(2+\frac{1}{2}\sqrt{3}\right)N$ and $\left(2-\frac{1}{2}\sqrt{3}\right)N$

(b) $\left(2+\sqrt{3}\right)N$ and $\left(2-\sqrt{3}\right)N$

(c) $\left(2+\frac{1}{2}\sqrt{2}\right)N$ and $\left(2-\frac{1}{2}\sqrt{2}\right)N$

(d) $\left(2+\sqrt{2}\right)N$ and $\left(2-\sqrt{2}\right)N$

117. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by **[2004]**

(a) 15 (b) 30 (c) 25 (d) 40

118. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in **[2003]**

(a) H.P (b) A.G.P (c) A.P (d) G.P.

119. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is least after a time **[2003]**

(a) $\frac{u\cos\alpha}{f}$ (b) $\frac{u\sin\alpha}{f}$ (c) $\frac{f\cos\alpha}{u}$ (d) $u\sin\alpha$

120. Two stones are projected from the top of a cliff h metres high, with the same speed u, so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan\theta$ equals **[2003]**

(a) $u\sqrt{\frac{2}{gh}}$ (b) $\sqrt{\frac{2u}{gh}}$ (c) $2g\sqrt{\frac{u}{h}}$ (d) $2h\sqrt{\frac{u}{g}}$

121. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by **[2003]**

(a) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$ (b) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$

(c) $\frac{2s}{\frac{1}{f}+\frac{1}{r}}$ (d) $\sqrt{2s(f+r)}$

122. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2$ is **[2003]**

(a) $2:3:1$ (b) $3:1:1$ (c) $2:3:2$ (d) $1:2:3$.

123. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle the moment of the couple thus formed is $\vec{H}$. If instead, the force $\vec{P}$ are turned through an angle α, then the moment of couple becomes **[2003]**

(a) $\vec{H}\sin\alpha-\vec{G}\cos\alpha$ (b) $\vec{G}\sin\alpha-\vec{H}\cos\alpha$

(c) $\vec{H}\sin\alpha+\vec{G}\cos\alpha$ (d) $\vec{G}\sin\alpha+\vec{H}\cos\alpha$.

124. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}-3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is **[2003]**

(a) 50 units (b) 20 units
(c) 30 units (d) 40 units.

125. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are **[2002]**

(a) $T=w\cos\theta$ $R=w\tan\theta$

(b) $T=2w\cos\theta$ $R=w$

(c) $T=w$ $R=w\sin\theta$

(d) $T=w\sin\theta$ $R=w\cot\theta$

126. The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are **[2002]**

(a) 13, 5 (b) 12, 6 (c) 14, 4 (d) 11, 7

Hints & Solutions

1. **(a)** $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right) = k$

$$a = \frac{k}{\cos\theta},\ b = \frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)},\ c = \frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

$$ab+bc+ca = k^2\frac{\left[\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)\right]}{\cos\left(\theta+\frac{4\pi}{3}\right)\cdot\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)}$$

$$= k^2\left[\frac{\cos\theta+2\cos(\theta+\pi)\cdot\cos\left(\frac{\pi}{3}\right)}{\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)\cdot\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{\cos\theta-2\cos\theta\cdot\frac{1}{2}}{\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)\cdot\cos\left(\theta+\frac{4\pi}{3}\right)}\right] = 0$$

$$\cos\phi = \frac{(a\hat{i}+b\hat{j}+c\hat{k})\cdot(b\hat{i}+c\hat{j}+a\hat{k})}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{b^2+c^2+a^2}}$$

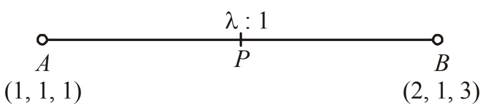
$$= ab+bc+ca = 0$$

$$\phi = \frac{\pi}{2}$$

2. **(0.8)**

Let position vector of A and B be $\vec{a}$ and $\vec{b}$ respectively.

$\therefore$ Position vector of P is $\overrightarrow{OP} = \frac{\lambda\vec{b}+\vec{a}}{\lambda+1}$



Given $\overrightarrow{OB}\cdot\overrightarrow{OP} - 3|\overrightarrow{OA}\times\overrightarrow{OP}|^2 = 6$

$$\Rightarrow \vec{b}\cdot\left(\frac{\lambda\vec{b}+\vec{a}}{\lambda+1}\right) - 3\left|\vec{a}\times\frac{\lambda\vec{b}+\vec{a}}{\lambda+1}\right|^2 = 6$$

$$\Rightarrow \frac{\vec{a}\cdot\vec{b}+\lambda|\vec{b}|^2}{\lambda+1} - \frac{3\lambda^2}{(\lambda+1)^2}|\vec{a}\times\vec{b}|^2 = 6$$

$(\because \vec{a}\times\vec{b} = 2\hat{i}-\hat{j}-\hat{k}$ and $\vec{a}\cdot\vec{b} = 6)$

$$\Rightarrow \frac{6+14\lambda}{\lambda+1} - \frac{18\lambda^2}{(\lambda+1)^2} = 6$$

$$\Rightarrow 6+\frac{8\lambda}{\lambda+1} - \frac{18\lambda^2}{(\lambda+1)^2} = 6$$

Let $\frac{\lambda}{\lambda+1} = t$

$$\Rightarrow 18t^2 - 8t = 0 \Rightarrow 2t(9t-4) = 0$$

$$\Rightarrow t = 0, \frac{4}{9}$$

$$\therefore \frac{\lambda}{\lambda+1} = \frac{4}{9} \Rightarrow \lambda = \frac{4}{5} = 0.8.$$

3. **(1.0)** $\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$

$$\Rightarrow 3a+1 = 0 \Rightarrow a = -\frac{1}{3}$$

The given vectors

$$\vec{p} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k} = \frac{1}{3}(2\hat{i}-\hat{j}-\hat{k})$$

$$\vec{q} = \frac{1}{3}(-\hat{i}+2\hat{j}-\hat{k})$$

$$\vec{r} = \frac{1}{3}(-\hat{i}-\hat{j}+2\hat{k})$$

Now, $\vec{p}.\vec{q} = \frac{1}{9}(-2-2+1) = -\frac{1}{3}$

$$\vec{r}\times\vec{q} = \frac{1}{9}\begin{vmatrix} i & j & k \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$$

$$= \frac{1}{9}(i(4-1) - j(-2-1) + k(1+2))$$

$$= \frac{1}{9}(3i+3j+3k) = \frac{i+j+k}{3}$$

$$|\vec{r}\times\vec{q}| = \frac{1}{3}\sqrt{3} \Rightarrow |\vec{r}\times\vec{q}|^2 = \frac{1}{3}$$

$$3(\vec{p}.\vec{q})^2 - \lambda|\vec{r}\times\vec{q}|^2 = 0$$

$$\Rightarrow 3.\frac{1}{9} - \lambda.\frac{1}{3} = 0 \Rightarrow \lambda = 1$$

4. **(b)** Let vector be $\lambda[(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})]$

Given, $a = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore$ $\vec{a}+\vec{b} = 4\hat{i} + 4\hat{j}$ and $\vec{a}-\vec{b} = 2\hat{i} + 4\hat{k}$

$\therefore$ vector $= \lambda[(4\hat{i}+4\hat{j})\times(2\hat{i}+4\hat{k})]$

$= \lambda[16\hat{i} - 16\hat{j} - 8\hat{k}] = 8\lambda[2\hat{i} - 2\hat{j} - \hat{k}]$

Given that magnitude of the vector is 12.

$\therefore$ $12 = 8|\lambda|\sqrt{4+4+1} \Rightarrow |\lambda| = \frac{1}{2}$

$\therefore$ required vector is $\pm 4\ (2\hat{i} - 2\hat{j} - \hat{k})$

5. **(b)** Volume of the parallelepiped is,

$$V = \left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\| = |1(1) + \lambda(\lambda^2) + 1(-\lambda)|$$

$= |\lambda^3 - \lambda + 1|$

Let $f(x) = x^3 - x + 1$

On differentiating, $f'(x) = 3x^2 - 1$

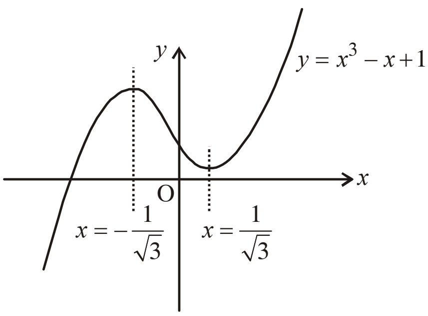
Now, $f'(x) = 0$

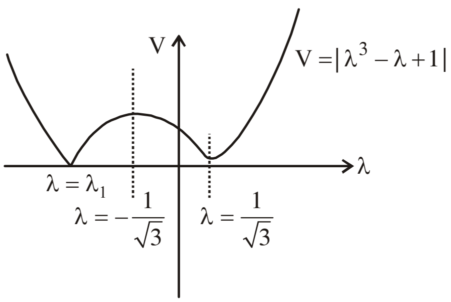
$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

and $f''(x) = 6x$

Since, $f''\left(\frac{1}{\sqrt{3}}\right) > 0$

$\therefore$ $x = \frac{1}{\sqrt{3}}$ is point of local minima.





For $\lambda = \lambda_1$, volume of parallelopiped is zero.

$\therefore$ vectors are coplanar.

6. **(b)** Let, three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar,

then $[\vec{a}, \vec{b}, \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0 \Rightarrow \alpha^2 + 6 = 0$$

$\because$ no real value of 'α' exist.

$\therefore$ set S is an empty set.

7. **(d)** Let $\cos\alpha$, $\cos\beta$, $\cos\gamma$ be direction cosines of a.

$\therefore$ $\cos\alpha = \cos\frac{\pi}{3}$, $\cos\beta = \cos\frac{\pi}{4}$ and $\cos\gamma = \cos\theta$

$\Rightarrow \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \pm\frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

8. **(a)** $\because$ Three vectors $(\mu\hat{i} + \hat{j} + \hat{k}), (\hat{i} + \mu\hat{j} + \hat{k}$ and $(\hat{i} + \hat{j} + \mu\hat{k})$ are copalnar.

$$\therefore \begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$\Rightarrow$ $\mu(\mu^2 - 1) + 1 - \mu + 1 - \mu = 0$

$\Rightarrow$ $(1-\mu)[2 - \mu(\mu+1)] = 0$

$\Rightarrow$ $(1-\mu)[\mu^2 + \mu - 2] = 0$

$\Rightarrow$ $\mu = 1, -2$

Therefore, sum of all real values $= 1 - 2 = -1$

9. **(d)** $\because$ $\bar{a}, \bar{b}$ and $\bar{c}$ are coplanar

$$\therefore \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & (\lambda^2 - 1) \end{vmatrix} = 0$$

$\Rightarrow$ $\lambda^3 - \lambda - 16 + 2(8 - \lambda^2 + 1) + 4(4 - 2\lambda) = 0$

$\Rightarrow$ $\lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

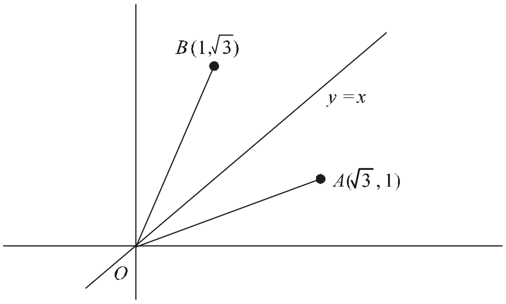
i.e., $(\lambda - 2)(\lambda - 3)(\lambda + 3) = 0$

For $\lambda = 2$, $\bar{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore \bar{a}\times\bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$$

For $\lambda = 3$ or -3, $\bar{c} = 2\bar{a} \Rightarrow \bar{a}\times\bar{c} = 0$ (Rejected)

10. (d) Since, the angle bisector of acute angle between OA and OB would be $y = x$



Since, the distance of C from bisector $= \frac{3}{\sqrt{2}}$

$$\Rightarrow \left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}} = 2\beta = \pm 3 + 1$$

$\beta = 2$ or $\beta = -1$

Hence, the sum of all possible value of $\beta = 2 + (-1) = 1$

11. (a) Let $\vec{\alpha}$ and $\vec{\beta}$ are collinear for same k

i.e., $\vec{\alpha} = k\vec{\beta}$

$(\lambda - 2)\ \vec{a} + \vec{b} = k((4\lambda - 2)\ \vec{a} + 3\vec{b})$

$(\lambda - 2)\ \vec{a} + \vec{b} = k(4\lambda - 2)\ \vec{a} + 3k\vec{b}$

$(\lambda - 2 - k(4\lambda - 2))\ \vec{a} + \vec{b}\ (1 - 3k) = 0$

But $\vec{a}$ and $\vec{b}$ are non-collinear, then

$\lambda - 2 - k(4\lambda - 2) = 0,\ 1 - 3k = 0$

$\Rightarrow\ k = \frac{1}{3}$ and $\lambda - 2 - \frac{1}{3}(4\lambda - 2) = 0$

$3\lambda - 6 - 4\lambda + 2 = 0$

$\lambda = -4$

12. (d) $\because \vec{u}, \vec{a}\ \&\ \vec{b}$ are coplanar

$\therefore \vec{u} = \lambda(\vec{a}\times\vec{b})\times\vec{a} \quad = \lambda\{\vec{a}^2.\vec{b} - (\vec{a}.\vec{b})\vec{a}\}$

$= \lambda\{-4\hat{i} + 8\hat{j} + 16\hat{k}\} = \lambda'\{-\hat{i} + 2\hat{j} + 4\hat{k}\}.$

Also, $\vec{u}.\vec{b} = 24 \ \Rightarrow \lambda' = 4$

$\therefore \quad \vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow \quad |\vec{u}|^2 = 336$

13. (c) Position vector of centriod $\vec{G} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$

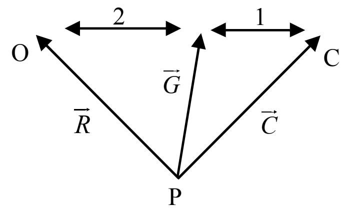
Position vector of circum centre $\vec{C} = \frac{\vec{a} + \vec{b} + \vec{c}}{4}$

$\vec{G} = \frac{2\vec{C} + \vec{r}}{3}$

$3\vec{G} = 2\vec{C} + \vec{r}$

$\vec{r} = 3\vec{G} - 2\vec{C} = (\vec{a} + \vec{b} + \vec{c}) - 2\left(\frac{\vec{a} + \vec{b} + \vec{c}}{4}\right)$

$= \frac{\vec{a} + \vec{b} + \vec{c}}{2}$

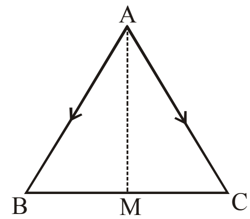


14. (c) We have,

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0 \Rightarrow \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

Let M be mid-point of BC

Now, $\overrightarrow{BM} = \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2} \quad \left(\because \ \overrightarrow{BM} = \frac{\overrightarrow{BC}}{2}\right)$



Also, we have

$\overrightarrow{AB} + \overrightarrow{BM} + \overrightarrow{MA} = 0$

$\Rightarrow \quad \overrightarrow{AB} + \frac{\overrightarrow{AC} - \overrightarrow{AB}}{2} = \overrightarrow{AM}$

$\Rightarrow \quad 1\,\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = 4\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow \quad \left|\overrightarrow{AM}\right| = \sqrt{33}$

15. (b) Since, $\vec{u}$ and $\vec{v}$ are collinear, therefore $k\vec{u} + \vec{v} = 0$

$\Rightarrow [k(\alpha - 2) + 2 + 3\alpha]\ \vec{a}\ + (k - 3)\ \vec{b}\ = 0$ ...(i)

Since $\vec{a}$ and $\vec{b}$ are non-collinear, then for some constant m and n,

$m\,\vec{a} + n\,\vec{b} = 0 \ \Rightarrow\ m = 0, n = 0$

Hence from equation (i)

$k - 3 = 0 \ \Rightarrow\ k = 3$

And $k(\alpha - 2) + 2 + 3\alpha = 0$

$\Rightarrow 3(\alpha - 2) + 2 + 3\alpha = 0 \ \Rightarrow\ \alpha = \frac{2}{3}$

16. (c) Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, and $\vec{c} = r\hat{i} + \hat{j} + (2r-1)\hat{k}$

Since, $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$ therefore, $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

$$\therefore \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ r & 1 & 2r-1 \end{vmatrix} = 0$$

$\Rightarrow 1\,(6r - 3 + 1) + 2\,(4r - 2 + r) + 3\,(2 - 3r) = 0$
$\Rightarrow 6r - 2 + 10r - 4 + 6 - 9r = 0$
$\Rightarrow r = 0$

17. (c) As per question

$$\vec{a} + 3\vec{b} = \lambda\vec{c} \qquad \text{....(i)}$$
$$\vec{b} + 2\vec{c} = \mu\vec{a} \qquad \text{....(ii)}$$

On solving equations (i) and (ii)

$$(1+3\mu)\vec{a} - (\lambda+6)\vec{c} = 0$$

As $\vec{a}$ and $\vec{c}$ are non collinear,

$\therefore \; 1 + 3\mu = 0$ and $\lambda + 6 = 0$

From (i), $\vec{a} + 3\vec{b} + 6\vec{c} = \vec{0}$

18. (d) The given vectors are coplanar then

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

$\Rightarrow \; p(qr-1) + 1(1-r) + 1(1-q) = 0$
$\Rightarrow \; pqr - p + 1 - r + 1 - q = 0$
$\Rightarrow \; pqr - (p+q+r) = -2$

19. (d) $\because \vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$

$\therefore \; \vec{a} = \vec{b} + \lambda\vec{c}$

$\Rightarrow \; \alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \hat{i} + \hat{j} + \lambda(\hat{j} + \hat{k})$

$\Rightarrow \; \alpha = 1, 2 = 1 + \lambda, \beta = \lambda$

$\Rightarrow \; \alpha = 1, \beta = 1$

ALTERNATE SOLUTION

$\because \vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$.

$\therefore \vec{a} = \lambda(\hat{b} + \hat{c})$

$$\Rightarrow \alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \frac{\lambda(\hat{i} + 2\hat{j} + \hat{k})}{\sqrt{2}}$$

$$\Rightarrow \alpha = \frac{\lambda}{\sqrt{2}}, \; \lambda = \sqrt{2}, \; \beta = \frac{\lambda}{\sqrt{2}}$$

$\Rightarrow \alpha = \beta = 1$

20. (d) If we consider unit vectors $\hat{i}$ and $\hat{j}$ in the direction AB and AC respectively and its magnitude $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively, then as per quesiton, forces along AB and AC respectively are

$$\left(\frac{1}{AB}\right)\hat{i} \text{ and } \left(\frac{1}{AC}\right)\hat{j}$$

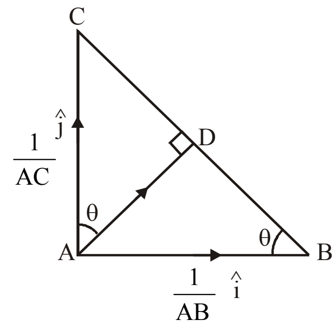
$\therefore$ Their resultant along $AD = \left(\frac{1}{AB}\right)i + \left(\frac{1}{AC}\right)j$

$\therefore$ Magnitude of resultant is

$$= \sqrt{\left(\frac{1}{AB}\right)^2 + \left(\frac{1}{AC}\right)^2} = \sqrt{\frac{AC^2 + AB^2}{AB^2 + AC^2}}$$

$[\because AC^2 + AB^2 = BC^2]$

$$= \frac{BC}{AB.AC}$$



$\because \; \Delta ABC \sim \Delta DBA$

$$\Rightarrow \frac{BC}{AB} = \frac{AC}{AD} \Rightarrow \frac{BC}{AB \times AC} = \frac{1}{AD}$$

$\therefore$ The required magnitude of resultant becomes $\frac{1}{AD}$.

21. (a) $\overrightarrow{PA} + \overrightarrow{AP} = 0$ and $\overrightarrow{PC} + \overrightarrow{CP} = 0$

$\Rightarrow \; \overrightarrow{PA} + \overrightarrow{AC} + \overrightarrow{CP} = 0 \quad \text{.... (i)}$

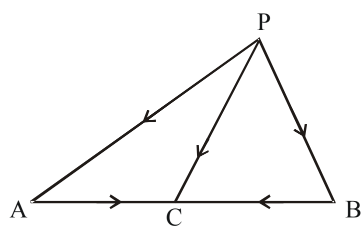
Similarly, $\overrightarrow{PB} + \overrightarrow{BC} + \overrightarrow{CP} = 0 \quad \text{.... (ii)}$

Adding eqn. (i) and (ii), we get

$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{AC} + \overrightarrow{BC} + 2\overrightarrow{CP} = 0.$

Since $\overrightarrow{AC} = -\overrightarrow{BC}$ & $\overrightarrow{CP} = -\overrightarrow{PC}$

$\Rightarrow \; \overrightarrow{PA} + \overrightarrow{PB} - 2\overrightarrow{PC} = 0.$



22. (a) Vector $a\vec{i} + a\vec{j} + c\vec{k}$, $\vec{i} + \vec{k}$ and $c\vec{i} + c\vec{j} + b\vec{k}$ are coplanar

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$$

$\therefore c$ is G.M. of a and b.

23. **(c)** If vectors $\vec{a}+2\vec{b}+3\vec{c}, \lambda\vec{b}+4\vec{c}$, and $(2\lambda-1)\vec{c}$ are coplanar then $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda-1 \end{vmatrix}=0$

$\Rightarrow \lambda(2\lambda-1)=0 \;\Rightarrow \lambda=0$ or $\frac{1}{2}$

$\therefore$ Forces are noncoplanar for all λ, except $\lambda=0, \frac{1}{2}$

24. **(c)** Given that $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$

Let $\vec{a}+2\vec{b}=t\vec{c}$ and $\vec{b}+3\vec{c}=s\vec{a}$, where t and s are scalars

$\therefore \vec{a}+2\vec{b}+6\vec{c}=t\vec{c}+6\vec{c}$

$=(t+6)\vec{c}$ [using $\vec{a}+2\vec{b}=t\vec{c}$]

$=\lambda\vec{c}$, where $\lambda=t+6$

25. **(none)** Given that

$A=(7,-4,7), B=(1,-6,10), C=(-1,-3,4)$

and $D=(5,-1,5)$

$\therefore \; AB=\sqrt{(7-1)^2+(-4+6)^2+(7-10)^2}$

$=\sqrt{36+4+9}=7$

Similarly, $BC=7, CD=\sqrt{41}, \; DA=\sqrt{17}$

$\therefore$ None of the options is satisfied.

26. **(c)** Given $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}+\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}=0$

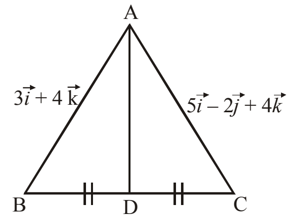
$\Rightarrow (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}=0$

Given that $(1, a, a^2), (1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar

$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}\neq 0$ (*given condition*)

$\therefore \; 1+abc=0 \Rightarrow abc=-1$

27. **(d)**



Given that AD is median of ΔABC.

$\therefore \; \overrightarrow{AD}=\frac{(3+5)i+(0-2)j+(4+4)k}{2}=4i-j+4k$

$|\overrightarrow{AD}|=\sqrt{16+16+1}=\sqrt{33}$

28. **(4)**

Let angle between $\vec{a}$ and $\vec{b}$ be θ.

$|\vec{a}+\vec{b}|=\sqrt{1+1+2\cos\theta}=2\left|\cos\frac{\theta}{2}\right|$ $[\because |a|=|b|=1]$

Similarly, $|\vec{a}-\vec{b}|=2\left|\sin\frac{\theta}{2}\right|$

So, $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|=2\left[\sqrt{3}\left|\cos\frac{\theta}{2}\right|+\left|\sin\frac{\theta}{2}\right|\right]$

$\because$ Maximum value of $(a\cos\theta+b\sin\theta)=\sqrt{a^2+b^2}$

$\therefore$ Maximum value $=2\sqrt{(\sqrt{3})^2+(1)^2}=4$.

29. **(1.00)**

$\because |\vec{x}+\vec{y}|=|\vec{x}|$

Squaring both sides we get

$|\vec{x}|^2+2\vec{x}.\vec{y}+|\vec{y}|^2=|\vec{x}|^2$

$\Rightarrow 2\vec{x}.\vec{y}+\vec{y}\cdot\vec{y}=0$ ...(i)

Also $2\vec{x}+\lambda\vec{y}$ and $\vec{y}$ are perpendicular

$\therefore 2\vec{x}\cdot\vec{y}+\lambda\vec{y}\cdot\vec{y}=0$ ...(ii)

Comparing (i) and (ii), $\lambda=1$

30. **(6.00)**

$\because$ Projection of $\vec{b}$ on $\vec{a}$ = Projection of $\vec{c}$ on $\vec{a}$

$\therefore \vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$

Given, $\vec{b}\cdot\vec{c}=0$

$\because |\vec{a}+\vec{b}-\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}\cdot\vec{b}-2\vec{b}\cdot\vec{c}-2\vec{a}\cdot\vec{c}$

$=4+16+16=36.$

$\Rightarrow |\vec{a}+\vec{b}-\vec{c}|^2=6$

31. **(2)**

$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2=8$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=-2$

Now, $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$

$=2|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2+4(\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c})=2$

32. **(8)** Let $P(1,-1,3), Q(2,-4,11), R(-1,2,3)$ and $S(3,-2,10)$

Then, $\overrightarrow{PQ}=\hat{i}-3\hat{j}+8\hat{k}$

Projection of $\overrightarrow{PQ}$ on $\overrightarrow{RS}$

$$=\frac{\overrightarrow{PQ}.\overrightarrow{RS}}{|\overrightarrow{RS}|}=\frac{4+12+56}{\sqrt{(4)^2+(4)^2+(7)^2}}=8$$

33. **(b)** It is given that $\vec{u}=\hat{i}+\hat{j}+\lambda\hat{k}$, $\vec{v}=\hat{i}+\hat{j}+3\hat{k}$ and $w=2\hat{i}+\hat{j}+\hat{k}$

Volume of parallelopiped $=[\vec{u}\,.\,\vec{v}\,.\,\vec{w}]$

$$\Rightarrow \pm1=\begin{vmatrix}1&1&\lambda\\1&1&3\\2&1&1\end{vmatrix}\Rightarrow -\lambda+3=\pm1\Rightarrow\lambda=2 \text{ or } \lambda=4$$

For $\lambda=2$

$$\cos\theta=\frac{2+1+2}{\sqrt{6}\sqrt{6}}=\frac{5}{6}$$

For $\lambda=4$

$$\cos\theta=\frac{2+1+4}{\sqrt{6}\sqrt{18}}=\frac{7}{6\sqrt{3}}$$

34. **(c)** Angle bisector between $\vec{b}$ and $\vec{c}$ can be

$\vec{a}=\lambda(\hat{b}+\hat{c})$ or $\vec{a}=\mu(\hat{b}-\hat{c})$

If $\vec{a}=\lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}+\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$$=\frac{\lambda}{3\sqrt{2}}[3\hat{i}+3\hat{j}+\hat{i}-\hat{j}+4\hat{k}]$$

$$=\frac{\lambda}{3\sqrt{2}}[4\hat{i}+2\hat{j}+4\hat{k}]$$

Compare with $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$$\frac{2\lambda}{3\sqrt{2}}=2\Rightarrow\lambda=3\sqrt{2}$$

$\vec{a}=4\hat{i}+2\hat{j}+4\hat{k}$

Not satisfy any option

Now consider $\vec{a}=\mu\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}-\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$$\vec{a}=\frac{\mu}{3\sqrt{2}}(3\hat{i}+3\hat{j}-\hat{i}+\hat{j}-4\hat{k})$$

$$=\frac{\mu}{3\sqrt{2}}(2\hat{i}+4\hat{j}-4\hat{k})$$

Compare with $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$$\frac{4\mu}{3\sqrt{2}}=2\Rightarrow\mu=\frac{3\sqrt{2}}{2}$$

$\vec{a}=\hat{i}+2\hat{j}-2\hat{k}$

$\therefore\ \vec{a}\,.\,\vec{k}+2=\left(\hat{i}+2\hat{j}-2\hat{k}\right)\cdot\hat{k}+2$

$=-2+2=0$

35. **(b)** $\because\ \vec{b}=2\vec{a}$

$\therefore\ 4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

$\therefore\ 3-\lambda_2=2\lambda_1$ ...(1)

$\because\ \vec{a}$ is perpendicular to $\vec{c}$

$\therefore\ \vec{a}\cdot\vec{c}=0$

$\Rightarrow\ 6+6\lambda_1+3(\lambda_3-1)=0$

$\Rightarrow\ 2+2\lambda_1+\lambda_3-1=0$

$\Rightarrow\ \lambda_3=-2\lambda_1-1$ ...(2)

Since $\left(\frac{-1}{2},4,0\right)$ satisfies equation (1) and (2). Hence, one of possible value of

$\lambda_1=-\frac{1}{2},\ \lambda_2=4$ and $\lambda_3=0$

36. **(b)** Projection of $\vec{b}$ on $\vec{a}$ $=\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{b_1+b_2+2}{4}$

According to question $\frac{b_1+b_2+2}{2}=\sqrt{1+1+2}=2$

$\Rightarrow\ b_1+b_2=2$ ...(1)

Since, $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$.

Hence, $\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}=0$

$\Rightarrow\ 8+5b_1+b_2+2=0$ ...(2)

From (1) and (2),

$b_1=-3,\ b_2=5$

$\Rightarrow\ \vec{b}=-3\cdot\hat{i}+5\hat{j}+\sqrt{2}\hat{k}$

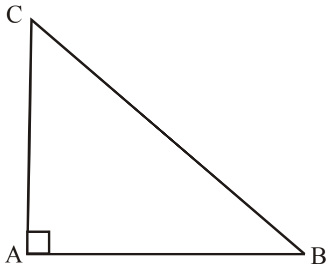
$|\vec{b}|=\sqrt{9+25+2}=6$

37. **(d)** $\overline{AB}=-4\hat{i}+2\hat{j}+(p+1)\hat{k}$

$\overline{AC}=2\hat{i}+(q-1)\hat{j}-3\hat{k}$

$\overline{AB}\perp\overline{AC}$

$\Rightarrow \quad \overline{AB}.\overline{AC} = 0$



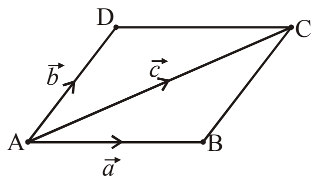
$-8 + 2(q-1) - 3(p+1) = 0$
$3p - 2q + 13 = 0$
(p, q) lies on $3x - 2y + 13 = 0$

$\text{slope} = \frac{3}{2}$

$\therefore$ Acute angle with x-axis

38. **(c)** Let $|\overrightarrow{AB}| = a$, $|\overrightarrow{AD}| = b$ and $|\overrightarrow{AC}| = c$

We have $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$



On squaring both the side, we get

$|\overrightarrow{AB}|^2 + |\overrightarrow{AD}|^2 + 2\,\overrightarrow{AB}.\overrightarrow{AD} = |\overrightarrow{AC}|^2$

$\Rightarrow a^2 + b^2 + 2\,\overrightarrow{AB}.(-\overrightarrow{DA}) = c^2$

$\Rightarrow 2\,\overrightarrow{AB}.\overrightarrow{DA} = a^2 + b^2 - c^2$

$\Rightarrow \overrightarrow{DA}.\overrightarrow{AB} = \frac{1}{2}(a^2 + b^2 - c^2)$

39. **(b)** $(\hat{x} + \hat{y} + \hat{z})^2 \ge 0$

$\Rightarrow \quad 3 + 2\Sigma\,\hat{x}.\hat{y} \ge 0$

$\Rightarrow \quad 2\Sigma\,\hat{x}.\hat{y} \ge -3$

Now, $|\hat{x} + \hat{y}|^2 + |\hat{y} + \hat{z}|^2 + |\hat{z} + \hat{x}|^2$

$= 6 + 2\Sigma\,\hat{x}.\hat{y} \ge 6 + (-3)$

$\Rightarrow \quad |\hat{x} + \hat{y}|^2 + |\hat{y} + \hat{z}|^2 + |\hat{z} + \hat{x}|^2 \ge 3$

40. **(c)** Given $|2\vec{a} - \vec{b}| = 5$

$\sqrt{(2|\vec{a}|)^2 + |\vec{b}|^2 - 2 \times |2\vec{a}||\vec{b}|\cos\theta} = 5$

Putting values of $|\vec{a}|$ and $|\vec{b}|$, we get

$(2 \times 2)^2 + (3)^2 - 24\cos\theta = 25$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

$|2\vec{a} + \vec{b}| = \sqrt{16 + 9 + 24\cos\theta} = \sqrt{25} = 5$

41. **(b)** Let angle between $\hat{a}$ and $\hat{c}$ be θ.

Now, $\hat{a} - \sqrt{3}\,\hat{b} + \hat{c} = \vec{0}$

$\Rightarrow (\hat{a} + \hat{c}) = \sqrt{3}\,\hat{b}$

$\Rightarrow \quad (\hat{a} + \hat{c}).(\hat{a} + \hat{c}) = 3(\hat{b}.\hat{b})$

$\Rightarrow \quad \hat{a}.\hat{a} + \hat{a}.\hat{c} + \hat{c}.\hat{a} + \hat{c}.\hat{c} = 3 \times 1$

$\Rightarrow \; 1 + 2\cos\theta + 1 = 3$

$\Rightarrow \cos\theta = \frac{1}{2} \;\; \Rightarrow \theta = \frac{\pi}{3}$

42. **(b)** Let $\vec{d} = \vec{b} + \lambda\,\vec{c}$

$\therefore \; \vec{d} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$

$= (1+\lambda)\hat{i} + (2+\lambda)\hat{j} - (1+2\lambda)\hat{k}$

If θ be the angle between $\vec{d}$ and $\vec{a}$, then projection of $\vec{d}$ or $(\vec{b} + \lambda\,\vec{c})$ on $\vec{a}$

$= |\vec{d}|\cos\theta = |\vec{d}|\left(\frac{\vec{d}.\vec{a}}{|\vec{d}||\vec{a}|}\right) = \frac{\vec{d}.\vec{a}}{|\vec{a}|}$

$= \frac{2(\lambda+1) - (\lambda+2) - (2\lambda+1)}{\sqrt{4+1+1}} = \frac{-\lambda-1}{\sqrt{6}}$

But projection of $\vec{d}$ on $\vec{a} = \sqrt{\frac{2}{3}}$

$\therefore \; -\frac{\lambda+1}{\sqrt{6}} = \sqrt{\frac{2}{3}} \Rightarrow \frac{\lambda^2 + 2\lambda + 1}{6} = \frac{2}{3}$

$\Rightarrow \lambda^2 + 2\lambda - 3 = 0 \;\Rightarrow\; \lambda^2 + 3\lambda - \lambda - 3 = 0$

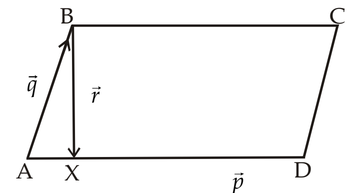
$\Rightarrow \lambda(\lambda+3) - 1(\lambda+3) = 0, \;\Rightarrow\; \lambda = 1, -3$

when $\lambda = 1$, then $\vec{b} + \lambda\,\vec{c} = 2\hat{i} + 3\hat{j} - 3\hat{k}$

when $\lambda = -3$, then $\vec{b} + \lambda\,\vec{c} = -2\hat{i} - \hat{j} + 5\hat{k}$

43. **(b)** Let $ABCD$ be a parallelogram such that $\overline{AB} = \vec{q}$, $\overline{AD} = \vec{p}$ and $\angle BAD$ be an acute angle.

We have



$\overline{AX} = \left(\frac{\vec{p}.\vec{q}}{|\vec{p}|}\right)\left(\frac{\vec{p}}{|\vec{p}|}\right) = \frac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

From triangle law

Let $\vec{r} = \overline{BX} = \overline{BA} + \overline{AX} = -\vec{q} + \frac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

44. **(c)** Given that $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ and $|\hat{a}| = |\hat{b}| = 1$

Since $\vec{c}$ and $\vec{d}$ are perpendicular to each other

$\therefore \quad \vec{c}\cdot\vec{d} = 0$

$\Rightarrow \quad (\hat{a} + 2\hat{b}).(5\hat{a} - 4\hat{b}) = 0$

$\Rightarrow \quad 5 + 6\hat{a}\cdot\hat{b} - 8 = 0$

$\Rightarrow \quad \hat{a}\cdot\hat{b} = \frac{1}{2} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

45. **(a)** Let $a + b + c = 0 \Rightarrow (a + b) = -c$

$\Rightarrow \quad (a + b)^2 = c^2$

$\Rightarrow \quad a^2 + b^2 + 2a.b = c^2$

$\Rightarrow \quad 9 + 25 + 2.3.5\cos\theta = 49$

$\left(\because \left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5 \text{ and } \left|\vec{c}\right| = 7\right)$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

46. **(d)** Let $x\hat{i} + y\hat{j} + z\hat{k}$ be the required unit vector.

Since $\hat{a}$ is perpendicular to $(2\hat{i} - \hat{j} + 2\hat{k})$.

$\therefore \quad 2x - y + 2z = 0$ .... (i)

Since vector $x\hat{i} + y\hat{j} + z\hat{k}$ is coplanar with the vector $\hat{i} + \hat{j} - \hat{k}$ and $2\hat{i} + 2\hat{j} - \hat{k}$.

$\therefore \quad x\hat{i} + y\hat{j} + z\hat{k} = p(\hat{i} + \hat{j} - \hat{k}) + q(2\hat{i} + 2\hat{j} - \hat{k})$,

where p and q are some scalars.

$\Rightarrow \quad x\hat{i} + y\hat{j} + z\hat{k} = (p + 2q)\hat{i} + (p + 2q)\hat{j} - (p + q)\hat{k}$

$\Rightarrow \quad x = p + 2q,\ y = p + 2q,\ z = -p - q$

Now from equation (i),

$2p + 4q - p - 2q - 2p - 2q = 0$

$\Rightarrow \quad -p = 0 \Rightarrow p = 0$

$\therefore \quad x = 2q,\ y = 2q,\ z = -q$

Since vector $x\hat{i} + y\hat{j} + z\hat{k}$ is a unit vector, therefore

$|x\hat{i} + y\hat{j} + z\hat{k}| = 1$

$\Rightarrow \quad \sqrt{x^2 + y^2 + z^2} = 1$

$\Rightarrow \quad x^2 + y^2 + z^2 = 1$

$\Rightarrow \quad 4q^2 + 4q^2 + q^2 = 1$

$\Rightarrow \quad 9q^2 = 1 \Rightarrow q = \pm\frac{1}{3}$

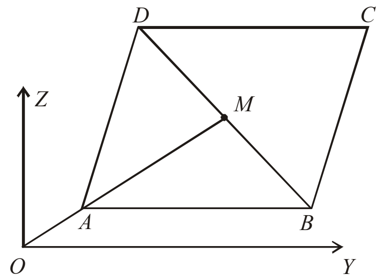
When $q = \frac{1}{3}$, then $x = \frac{2}{3}, y = \frac{2}{3}, z = -\frac{1}{3}$

When $q = -\frac{1}{3}$, then $x = -\frac{2}{3}, y = -\frac{2}{3}, z = \frac{1}{3}$

Here required unit vector is $\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$

or $-\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$.

47. **(d)**



In a parallelogram, diagonals bisect each other. So, mid point of DB is also the mid-point of AC.

Mid-point of $M = 2\hat{i} - \hat{j}$

Direction ratio of $OC = (1, -5, -5)$

Direction ratio of $OM = (2, -1, 0)$

Angle θ between OM and OC is given by

$$\cos\theta = \frac{(1\times 2) + (-5)(-1) + (-5)(0)}{\sqrt{2^2 + (-1)^2}\sqrt{(1)^2 + (-5)^2 + (-5)^2}}$$

$$= \frac{2 + 5}{\sqrt{5}\sqrt{51}} = \frac{7}{\sqrt{5}\sqrt{51}}$$

Projection of $\overrightarrow{OM}$ on $\overrightarrow{OC}$ is given by

$|OM|.\cos\theta = \sqrt{5} \times \frac{7}{\sqrt{5}\times\sqrt{51}} = \frac{7}{\sqrt{51}}$

48. **(d)** Given that, $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually orthogonal

$\therefore \vec{a}.\vec{b} = 0,\ \vec{b}.\vec{c} = 0,\ \vec{c}.\vec{a} = 0$

$\Rightarrow 2\lambda + 4 + \mu = 0$ ...(i)

$\lambda - 1 + 2\mu = 0$ ...(ii)

On solving (i) and (ii), we get $\lambda = -3, \mu = 2$

49. **(d)** Clearly $\vec{a} = -\frac{8}{7}\vec{c}$

$\Rightarrow \vec{a} \parallel \vec{c}$ and are opposite in direction

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

50. **(a)** $\overline{CA} = (2 - a)\hat{i} + 2\hat{j}$; $\overline{CB} = (1 - a)\hat{i} - 6\hat{k}$

$[\because \overline{CA} \perp \overline{CB}]$

$\therefore \quad \overline{CA}\cdot\overline{CB} = 0 \Rightarrow (2 - a)(1 - a) = 0$

$\Rightarrow \quad a = 2, 1$

51. **(c)** Projection of $\vec{v}$ along $\vec{u} = \frac{\vec{v}.\vec{u}}{|\vec{u}|} = \vec{v}.\vec{u}$

projection of $\vec{w}$ along $\vec{u} = \frac{\vec{w}.\vec{u}}{|\vec{u}|} = \vec{w}.\vec{u}$

Given $\vec{v}.\vec{u} = \vec{w}.\vec{u}$ ....(1)

Also , $\vec{v}.\vec{w}=0$ [$\because$ $\vec{v}\perp\vec{w}$] ....(2)

Now $|\vec{u}-\vec{v}+\vec{w}|^2$

$=|\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2-2\vec{u}.\vec{v}-2\vec{v}.\vec{w}+2\vec{u}.\vec{w}$

$=1+4+9+0$ [From (1) and (2)] $=14$

$\therefore |\vec{u}-\vec{v}+\vec{w}|=\sqrt{14}$

52. **(c)** Given that $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$1+4+9+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=\dfrac{-1-4-9}{2}=-7$

53. **(a)** Given that, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2|+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow 25+16+9+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-25\cdot$

$\therefore |\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|=25\cdot$

54. **(a)** Given that $\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow |\vec{b}+\vec{c}|^2=|\vec{a}|^2=5^2+3^2+2\vec{b}\cdot\vec{c}=7^2$

$\Rightarrow 2|\vec{b}||\vec{c}|\cos\theta=49-34=15$;

$\Rightarrow 2\times 5\times 3\cos\theta=15$;

$\Rightarrow \cos\theta=1/2; \Rightarrow \theta=\dfrac{\pi}{3}=60°$

55. **(b)** We know that the volume of parallelopiped

$=[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$\begin{vmatrix}1 & 1 & n\\ 2 & 4 & -n\\ 1 & n & 3\end{vmatrix}=158$

$\Rightarrow (12+n^2)-1(6+n)+n(2n-4)=158$

$\Rightarrow 3n^2-5n-152=0$

$\Rightarrow 3n^2-24n+19n-152=0$

$\Rightarrow 3n(n-8)+19(n-8)=0$

$\Rightarrow n=8$ or $n=\dfrac{-19}{3}$

$\Rightarrow n=8$ $\quad(\because n\geq 0)$

$\therefore \vec{a}=\hat{i}+\hat{j}+8\hat{k},\ \vec{b}=2\hat{i}+4\hat{j}-8\hat{k}$ and

$\vec{c}=\hat{i}+8\hat{j}+3\hat{k}$

$\vec{a}\cdot\vec{c}=1+8+24=33$

$\vec{b}\cdot\vec{c}=2+32-24=10$

56. **(d)** It is given that

$$f(x)=\vec{a}\cdot(\vec{b}\times\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & -2 & 3\\ -2 & x & -1\\ 7 & -2 & x\end{vmatrix}=x^3-27x+26$$

$\Rightarrow f(x)=x^3-27x+26$

$\Rightarrow f'(x)=3x^2-27$

For critical point $f'(x)=0$

$\Rightarrow 3x^2-27=0 \Rightarrow x=-3,\ 3$

+ − +
−3 3
Max. Min.

The local maxima of $f(x)$ is, $x_0=-3$.

Then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$

$=-2x-2x-3-14-2x-x+7x+4+3x=3x-13$

So, value at $x=x_0,=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=3x-13$

$=3\times(-3)-13=-22.$

57. **(18)**

$\hat{i}\times(\vec{a}\times\hat{i})=(\hat{i}\cdot\hat{i})\vec{a}-(\hat{i}\cdot\vec{a})\hat{i}=\hat{j}+2\hat{k}$

Similarly, $\hat{j}\times(\vec{a}\times\hat{j})=2\hat{i}+2\hat{k}$,

$\hat{k}\times(\vec{a}\times\hat{k})=2\hat{i}+\hat{j}$

$\therefore |\hat{j}+2\hat{k}|^2+|2\hat{i}+2\hat{k}|^2+|2\hat{i}+\hat{j}|^2=5+8+5=18.$

58. **(30)** $\vec{b}.\vec{c}=10 \Rightarrow |\vec{b}|\ |\vec{c}|\cos\left(\dfrac{\pi}{3}\right)=10$

$\Rightarrow 5.|\vec{c}|.\dfrac{1}{2}=10 \Rightarrow |\vec{c}|=4$

Since, is perpendicular to the vector $\vec{b}\times\vec{c}$, then

$\vec{a}.(\vec{b}\times\vec{c})=0$

Now, $|\vec{a}\times(\vec{b}\times\vec{c})|=|\vec{a}||\vec{b}\times\vec{c}|\sin\left(\dfrac{\pi}{2}\right)$

$=\sqrt{3}\times|\vec{b}||\vec{c}|\sin\dfrac{\pi}{3}\times 1$

Hence, $|\vec{a}\times(\vec{b}\times\vec{c})|=30.$

59. **(c)** $\vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times(\vec{b}\times\vec{a})$

$\Rightarrow -(\vec{a}.\vec{b})\vec{c}=(\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a}$

$\Rightarrow \quad -4\vec{c} = 6(\hat{i} - \hat{j} + \hat{k}) - 4(\hat{i} - 2\hat{j} - \hat{k})$

$\Rightarrow \quad -4\vec{c} = 2\hat{i} - 2\hat{j} + 2\hat{k}$

$\Rightarrow \quad \vec{c} = \frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$

$\Rightarrow \quad \vec{b}.\vec{c} = -\frac{1}{2}$

60. (d) $\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = 0$

$3 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = \frac{-3}{2} \Rightarrow \lambda = \frac{-3}{2}$

$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times (-\vec{a} - \vec{b}) + (-\vec{a} - \vec{b}) \times \vec{a} \quad [\because \vec{c} = -\vec{a} - \vec{b}]$

$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b}$

$\vec{d} = 3(\vec{a} \times \vec{b})$

61. (c) $\vec{\beta} = \vec{\beta_1} - \vec{\beta_2}$ ...(1)

Since, $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

$\therefore \vec{\beta_2}.\vec{\alpha} = 0$

Since, $\vec{\beta_1}$ is parallel to $\vec{a}$.

then $\vec{\beta_1} = \lambda\vec{\alpha}$ (say)

$\vec{a}.\vec{\beta} = \vec{a}.\vec{\beta_1} - \vec{\alpha}.\vec{\beta_2}$

$\Rightarrow 5 = \lambda\alpha^2 \Rightarrow 5 = \lambda \times 10 \qquad \left(\because |\vec{\alpha}| = \sqrt{10}\right).$

$\Rightarrow \lambda = \frac{1}{2} \qquad \therefore \vec{B_1} = \frac{\vec{\alpha}}{2}$

$\therefore \vec{\beta_1} = \frac{\vec{\alpha}}{2}$

Cross product with $\vec{B_1}$ in equation (1)

$\Rightarrow \vec{\beta} \times \vec{B_1} = -\vec{B_2} \times \vec{B_1}$

$\Rightarrow \vec{\beta} \times \vec{B_1} = \vec{B_1} \times \vec{B_2} \Rightarrow \vec{\beta_1} \times \vec{\beta_2} = \frac{\vec{\beta} \times \vec{\alpha}}{2}$

$\Rightarrow \vec{B_1} \times \vec{B2} = \frac{1}{2}\begin{vmatrix} \hat{i} & j & k \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$

$= \frac{1}{2}\left[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)\right] \qquad = \frac{1}{2}\left[-3\hat{i} + 9\hat{j} + 5\hat{k}\right]$

62. (d) Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$

$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$

Now, projection of vector $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ on $\vec{a} \times \vec{b}$ is

$= \left|\frac{\vec{c}.(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}\right| = \left|\frac{2 - 6 + 1}{\sqrt{6}}\right| = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$

63. (b) Given, $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$= (2 + x)\hat{i} + (x - 3)\hat{j} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(2 + x)^2 + (x - 3)^2 + (-5)^2} = r$

$\Rightarrow r = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$

$= \sqrt{2x^2 - 2x + 38} = \sqrt{2\left(x^2 - x + \frac{1}{4}\right) + 38 - \frac{1}{2}}$

$= \sqrt{2\left(x - \frac{1}{2}\right)^2 + \frac{75}{2}} \Rightarrow r \geq \sqrt{\frac{75}{2}} \Rightarrow r \geq 5\sqrt{\frac{3}{2}}$

64. (a) Since, $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors

$\therefore \quad |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Then, $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$

$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{2}\vec{b}$

$\therefore \quad \vec{a} \cdot \vec{c} = \frac{1}{2}$ and $\vec{a} \cdot \vec{b} = 0$

$|\vec{a}||\vec{c}|\cos\beta = \frac{1}{2}$ and $|\vec{a}||\vec{b}|\cos\alpha = 0$

$\Rightarrow \quad \beta = 60°$ and $\alpha = 90°$

Hence, $|\alpha - \beta| = |90° - 60°| = 30°$

65. (a) $\because |\vec{a} \times \vec{c}|^2 = |\vec{a}|^2|\vec{c}|^2 - (\vec{a}.\vec{c})^2$

$\Rightarrow \quad |-\vec{b}|^2 = 2|\vec{c}|^2 - 16 \Rightarrow 3 = 2|\vec{c}|^2 - 16$

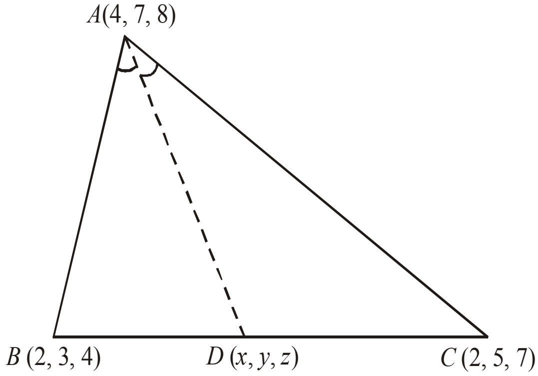
$\Rightarrow \quad |\vec{c}|^2 = \frac{19}{2}$

66. **(b)** Suppose angular bisector of A meets BC at D (x, y, z)
Using angular bisector theorem,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{BD}{DC} = \frac{\sqrt{(4-2)^2 + (7-3)^2 + (8-4)^2}}{\sqrt{(4-2)^2 + (7-5)^2 + (8-7)^2}}$$

$$= \frac{\sqrt{2^2 + 4^2 + 4^2}}{\sqrt{2^2 + 2^2 + 1^2}} = \frac{6}{3} = 2$$



So, $D(x, y, z) \equiv \left(\frac{(2)(2)+(1)(2)}{2+1}, \frac{(2)(5)+(1)(3)}{2+1}, \frac{(2)(7)+(1)(4)}{2+1}\right)$

$$D(x, y, z) \equiv \left(\frac{6}{3}, \frac{13}{3}, \frac{18}{3}\right)$$

Therefore, position vector of point $P = \frac{1}{3}(6i + 13j + 18k)$

67. **(a)** $\because \ \vec{a} = \hat{i} + \hat{j} + \hat{k} \Rightarrow |\vec{a}| = \sqrt{3}$

& $\vec{c} = \hat{j} - \hat{k} \Rightarrow |\vec{c}|\sqrt{2}$

Now, $\vec{a} \times \vec{b} = \vec{c}$ (Given)

$\Rightarrow |\vec{a}||\vec{b}| \sin\theta = |\vec{c}|$

$\Rightarrow |\vec{a}||\vec{b}| \sin\theta = \sqrt{2}$ ...[i]

Also $\vec{a} \cdot \vec{b} = 3$

$\Rightarrow |\vec{a}||\vec{b}| \cos\theta = 3$ ....[ii]

Dividing [i] by [ii], we get

$\tan\theta = \frac{\sqrt{2}}{3} \ \therefore \ \sin\theta = \frac{\sqrt{2}}{\sqrt{11}}$

Substituting value of sinθ in [i] we get

$$\sqrt{3}|\vec{b}|\frac{\sqrt{2}}{\sqrt{11}} = \sqrt{2}$$

$$|\vec{b}| = \frac{\sqrt{11}}{\sqrt{3}}$$

68. **(b)** $\because \vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$ [Given]

$\Rightarrow \vec{a} + 2\vec{c} = -2\vec{b} \Rightarrow (\vec{a} + 2\vec{c}) \cdot (\vec{a} + 2\vec{c}) = (-2\vec{b})(-2\vec{b})$

$\Rightarrow \vec{a} \cdot \vec{a} + 4\vec{c} \cdot \vec{c} + 4\vec{a} \cdot \vec{c} = 4\vec{b} \cdot \vec{b} \Rightarrow 1 + 4 + 4\vec{a} \cdot \vec{c} = 4$

$\Rightarrow \vec{a} \cdot \vec{c} = \frac{-1}{4}$

$\because |\vec{a} \cdot \vec{c}|^2 + |\vec{a} \times \vec{c}|^2 = 1$ ($\vec{a}$ is unit vector)

$\Rightarrow \frac{1}{16} + |\vec{a} \times \vec{c}|^2 = 1$

$\Rightarrow |\vec{a} \times \vec{c}|^2 = \frac{15}{16} \Rightarrow |\vec{a} \times \vec{c}| = \frac{\sqrt{15}}{4}$

69. **(c)** Given : $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \ \vec{b} = \hat{i} + \hat{j}$

$\Rightarrow |\vec{a}| = 3$

$\therefore \ \vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{2^2 + 2^2 + 1^2} = 3$

We have $(\vec{a} \times \vec{b}) \times \vec{c} = |\vec{a} \times \vec{b}||\vec{c}| \sin 30 \ n$

$\Rightarrow |(\vec{a} \times \vec{b}) \times \vec{c}| = 3|\vec{c}|.\frac{1}{2} \Rightarrow 3 = 3|\vec{c}|.\frac{1}{2}$

$\therefore \ |\vec{c}| = 2$

Now $|\vec{c} - \vec{a}| = 3$

On squaring, we get

$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{c} = 9 \Rightarrow 4 + 9 - 2\,\vec{a}.\vec{c} = 9$

$\Rightarrow \vec{a}.\vec{c} = 2 \ [\because \vec{c}.\vec{a} = \vec{a}.\vec{c}]$

70. **(b)** $\vec{b_1} = \frac{(\vec{b_1} \cdot \vec{a})\hat{a}}{1} = \left\{\frac{(3\hat{j} + 4\hat{k}).(\hat{i} + \hat{j})}{\sqrt{2}}\right\}\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

$= \frac{3(\hat{i} + \hat{j})}{\sqrt{2} \times \sqrt{2}} = \frac{3(\hat{i} + \hat{j})}{2}$

$\vec{b_1} + \vec{b_2} = \vec{b}$

$\Rightarrow \vec{b_2} = \vec{b} - \vec{b_1} = (3\hat{j} + 4\hat{k}) - \frac{3}{2}(\hat{i} + \hat{j})$

$\Rightarrow \boxed{\vec{b_2} = -\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + 4\hat{k}}$

& $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{3}{2} & 0 \\ -\frac{3}{2} & \frac{3}{2} & 4 \end{vmatrix}$

$\Rightarrow \vec{b_1} \times \vec{b_2} = \hat{i}(6) - \hat{j}(6) + \hat{k}\left(-\frac{9}{4} + \frac{9}{4}\right)$

$\Rightarrow 6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

71. (b) Let; $d_1 = 8\hat{i} - 6\hat{j} + 0\hat{k}$ & $d_2 = 3\hat{i} + 4\hat{j} - 12\hat{k}$

$\therefore \quad |d_1 \times d_2| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix} = |72\hat{i} - (-96)\hat{j} + 50\hat{k}|$

$\Rightarrow \quad |d_1 \times d_2| = \sqrt{16900} = 130$

$\therefore$ Area of parallelogram $= \frac{1}{2}|d_1 \times d_2| = \frac{1}{2} \times 130 = 65$

72. (b) $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$

$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$

On comparing both sides

$\vec{a} \cdot \vec{b} = -\frac{\sqrt{3}}{2} \Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$

[$\because \vec{a}$ and $\vec{b}$ are unit vectors]

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\theta = \frac{5\pi}{6}$

73. (c) $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \quad -\vec{c} \times (\vec{a} \times \vec{b}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \quad -(\vec{c}.\vec{b})\vec{a} + (\vec{c}.\vec{a})\vec{b} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \quad -|\vec{b}||\vec{c}|\cos\theta\vec{a} + (\vec{c}.\vec{a})\vec{b} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\because \quad \vec{a}, \vec{b}, \vec{c}$ are non collinear, the above equation is possible only when

$-\cos\theta = \frac{1}{3}$ and $\vec{c}.\vec{a} = 0$

$\Rightarrow \quad \cos\theta = -\frac{1}{3}$

$\Rightarrow \quad \sin\theta = \frac{2\sqrt{2}}{3}$; $\theta \in$ II quad

74. (a) $|\vec{a} + \vec{b}| = \sqrt{3}$

angle between $\vec{a}$ and $\vec{b}$ is 60°.

$\vec{a} \times \vec{b}$ is $\perp^r$ to plane containing $\vec{a}$ and $\vec{b}$

$\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$

$\vec{c} = \sqrt{|a|^2 + 4|\vec{b}|^2 + 2.2|\vec{a}|^2 \cos 60° \vec{n}_1} + 3|\vec{a}||\vec{b}|\sin 60° \vec{n}_2$

$+3|\vec{a}||\vec{b}|\sin 60°.\vec{n}_2$

$\vec{n}_1 \perp^r \vec{n}_2$

$|\vec{c}|^2 = (1+4+2) + 9 \times \frac{3}{4} \Rightarrow |\vec{c}|^2 = 7 + 27/4 = 55/4$

$2|\vec{c}| = \sqrt{55}$

75. (b) L.H.S $= (\vec{a} \times \vec{b}).[(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})]$

$= (\vec{a} \times \vec{b}).[(\vec{b} \times \vec{c}.\vec{a})\vec{c} - (\vec{b} \times \vec{c}.\vec{c})\vec{a}]$

$= (\vec{a} \times \vec{b}).[[\vec{b}\,\vec{c}\,\vec{a}]\vec{c}] \quad [\because \vec{b} \times \vec{c}.\vec{c} = 0]$

$= [\vec{a}\,\vec{b}\,\vec{c}].(\vec{a} \times \vec{b}.\vec{c}) = [\vec{a}\,\vec{b}\,\vec{c}]^2$

$= [\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$

So $\lambda = 1$

76. (c) Let $\vec{x} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} - 12\hat{k}$

Now, $\vec{x} \times \vec{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -1 \\ 1 & 4 & -3 \end{vmatrix} = 22\hat{i} + 8\hat{j} + 18\hat{k}$

Projection of $\vec{x} \times \vec{y}$ on $\vec{z} = \frac{(\vec{x} + \vec{y}).(\vec{z})}{|\vec{z}|}$

$= \frac{22(3) + 8(-4) + 18(-12)}{\sqrt{9 + 16 + 144}} = \frac{-182}{13} = -14$

Now, magnitude of projection = 14.

77. (d) Let, $\vec{c} = a\hat{i} + b\hat{j} + c\hat{k}$

Given, $\vec{c} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = \vec{0}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 1 & 2 & 5 \end{vmatrix} = \vec{0}$

$\Rightarrow (5b - 2c)\hat{i} - (5a - c)\hat{j} + (2a - b)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Comparing both sides, we get

$5b - 2c = 0$; $5a - c = 0$; $2a - b = 0$

or $5b = 2c$; $5a = c$; $2a = b$

Also given $|\vec{c}|^2 = 60 \Rightarrow a^2 + b^2 + c^2 = 60$

Putting the value of b and c in above eqn., we get

$a^2 + (2a)^2 + (5a)^2 = 60$

$\Rightarrow a^2 + 4a^2 + 25a^2 = 60 \Rightarrow 30a^2 = 60$

$a^2 = 2$

$a = \pm\sqrt{2}$; $b = 2\sqrt{2}$; $c = 5\sqrt{2}$

Now, $\vec{c} = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore \vec{c} = \sqrt{2}\hat{i} + 2\sqrt{2}\hat{j} + 5\sqrt{2}\hat{k}$

Value of $\vec{c}.(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is

$(\sqrt{2}\hat{i} + 2\sqrt{2}\hat{j} + 5\sqrt{2}\hat{k}).(-7\hat{i} + 2\hat{j} + 3\hat{k})$

$= -7\sqrt{2} + 4\sqrt{2} + 15\sqrt{2} = 12\sqrt{2}$

78. (d) $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$

$\Rightarrow |\vec{a}| = 3$

and $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{4+4+1} = 3$

Now, $|\vec{c}-\vec{a}| = 2\sqrt{2} \Rightarrow |\vec{c}-\vec{a}|^2 = 8$

$\Rightarrow |\vec{c}-\vec{a}|.(\vec{c}-\vec{a}) = 8$

$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}.\vec{a} = 8$

$\Rightarrow |\vec{c}|^2 + 9 - 2|\vec{c}| = 8$

$\Rightarrow (|\vec{c}|-1)^2 = 0 \Rightarrow |\vec{c}| = 1$

$\therefore |(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}|\sin 30° = 3\times1\times\frac{1}{2} = \frac{3}{2}$

79. **(c)** $(\hat{i}\times\vec{a}\cdot\vec{b})\hat{i} + (\hat{j}\times\vec{a}\cdot\vec{b})\hat{j} + (\hat{k}\times\vec{a}\cdot\vec{b})\hat{k}$

$= (\hat{i}\cdot\vec{a}\times\vec{b})\hat{i} + (\hat{j}\cdot\vec{a}\times\vec{b})\hat{j} + (\hat{k}\cdot\vec{a}\times\vec{b})\hat{k}$

$(\because \vec{a}\times\vec{b}\cdot\vec{c} = \vec{a}\cdot\vec{b}\times\vec{c})$

$= (\vec{a}\times\vec{b})\hat{i} + (\vec{a}\times\vec{b})\hat{j} + (\vec{a}\times\vec{b})\hat{k} = \vec{a}\times\vec{b}$

80. **(c)** Statement - 1

The vectors $\vec{a},\vec{b}$ and $\vec{c}$ lie in the same plane.

$\Rightarrow \vec{a},\vec{b}$ and $\vec{c}$ are coplanar.

We know, the necessary and sufficient conditions for three vectors to be coplanar is that $[\vec{a}\vec{b}\vec{c}] = 0$

i.e. $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0$

Hence, statement-1 is true.

81. **(b)** Let $\vec{u} = \hat{j}+4\hat{k}$, $\vec{v} = \hat{i}-3\hat{k}$ and

$\vec{w} = \cos\theta\hat{i} + \sin\theta\hat{j}$

Now, $\vec{u}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 4 \\ 1 & 0 & -3 \end{vmatrix} = \hat{i}(-3) - \hat{j}(-4) + \hat{k}(-1)$

$= -3\hat{i} + 4\hat{j} - \hat{k}$

Now, $(\vec{u}\times\vec{v})\cdot\vec{w} = (-3\hat{i}+4\hat{j}-\hat{k}).(\cos\theta\hat{i}+\sin\theta\hat{j})$

$= -3\cos\theta + 4\sin\theta$

Now, maximum possible value of

$|-3\cos\theta + 4\sin\theta| = \sqrt{(-3)^2+(4)^2} = \sqrt{25} = 5$

82. **(a)** **Statement - 1**

Points (1, 2, 2), (2, 1, 2), (2, 2, z) and (1, 1, 1) are coplanar then $z = 2$ which is false.

$\because \begin{vmatrix} 1 & -1 & 0 \\ 1 & 0 & z-2 \\ 0 & -1 & -1 \end{vmatrix} = 0$

$\Rightarrow 1(z-2) + 1(-1) = 0 \Rightarrow z = 3$

Statement - 2 is the true statement.

83. **(a)** Since $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and

$\vec{c} = \lambda\hat{i} + \hat{j} + (2\lambda-1)\hat{k}$ are coplanar

therefore $[\vec{a}\,\vec{b}\,\vec{c}] = 0$

i.e., $\begin{vmatrix} 1 & 2 & \lambda \\ -2 & 3 & 1 \\ 3 & -1 & 2\lambda-1 \end{vmatrix} = 0$

$\Rightarrow 1(6\lambda-2) - 2(-4\lambda-1) + \lambda(-7) = 0$

$\Rightarrow (6\lambda-2) + 8\lambda + 2 + 2 + 2\lambda - 9\lambda = 0$

$\Rightarrow 7\lambda = 0 \Rightarrow \lambda = 0$

84. **(c)** Given that $\vec{a}.\vec{b} \neq 0$, $\vec{a}.\vec{d} = 0$

Now, $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{b}\times\vec{c}) = \vec{a}\times(\vec{b}\times\vec{d})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{d})\vec{b} - (\vec{a}.\vec{b})\vec{d}$

$\Rightarrow (\vec{a}.\vec{b})\vec{d} = -(\vec{a}.\vec{c})\vec{b} + (\vec{a}.\vec{b})\vec{c}$

$\vec{d} = \vec{c} - \left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

85. **(d)** $(2\vec{a}-\vec{b}).((\vec{a}\times\vec{b})\times(\vec{a}+2\vec{b}))$

$= (2\vec{a}-\vec{b})\cdot((\vec{a}\times\vec{b})\times\vec{a} + 2(\vec{a}\times\vec{b})\times\vec{b})$

$= (2\vec{a}-\vec{b})((\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a} + 2(\vec{a}.\vec{b})\vec{b} - 2(\vec{b}.\vec{b})\vec{a})$

$= (2\vec{a}-\vec{b})(\vec{b} - 0 + 0 - 2\vec{a})$

From given values we get

$\vec{a}.\vec{b} = 0$ and $\vec{b}.\vec{b} = 1$

$= -4\vec{a}.\vec{a} - \vec{b}.\vec{b} = -5$

86. **(d)** Given that

$\vec{c} = \vec{b}\times\vec{a} \Rightarrow \vec{b}.\vec{c} = \vec{b}.(\vec{b}\times\vec{a}) \Rightarrow \vec{b}.\vec{c} = 0$

$\Rightarrow (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}).(\hat{i} - \hat{j} - \hat{k}) = 0,$

where $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$b_1 - b_2 - b_3 = 0$ ...(i)

and $\vec{a}.\vec{b} = 3 \Rightarrow (\hat{j}-\hat{k}).(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 3$

$\Rightarrow b_2 - b_3 = 3$

From equation (i)

$b_1 = b_2 + b_3 = (3+b_3) + b_3 = 3 + 2b_3$

$\vec{b} = (3+2b_3)\hat{i} + (3+b_3)\hat{j} + b_3\hat{k}$

From the option given, it is clear that b_3 equal to either 2 or -2.

If $b_3 = 2$ then $\vec{b} = 7\hat{i} + 5\hat{j} + 2\hat{k}$ which is not possible

If $b_3 = -2$, then $\vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

87. **(d)** $\because \vec{u}, \vec{v}, \vec{w}$ are non coplanar vectors

$\therefore [\vec{u}, \vec{v}, \vec{w}] \neq 0$

Now, $[3\vec{u}, p\vec{v}, p\vec{w}] - [p\vec{v}, p\vec{w}, q\vec{u}] - [2\vec{w}, q\vec{v}, q\vec{u}] = 0$

$\Rightarrow\ 3p^2[\vec{u},\vec{v},\vec{w}]-pq[\vec{v},\vec{w},\vec{u}]-2q^2[\vec{w},\vec{v},\vec{u}]=0$

$\Rightarrow\ 3p^2[\vec{u},\vec{v},\vec{w}]-pq[\vec{u},\vec{v},\vec{w}]+2q^2[\vec{u},\vec{v},\vec{w}]$

$\Rightarrow\ (3p^2-pq+2q^2)\ [\vec{u},\vec{v},\vec{w}]=0$

$\Rightarrow\ 3p^2-pq+2q^2=0$
$(\because [\vec{u},\vec{v},\vec{w}]=0)$

$\Rightarrow\ 2p^2+p^2-pq+\frac{q^2}{4}+\frac{7q^2}{4}=0$

$\Rightarrow\ 2p^2+\left(p-\frac{q}{2}\right)^2+\frac{7}{4}q^2=0$

$\Rightarrow\ p=0, q=0, p=q/2$

This is possible only when $p=0, q=0$

$\therefore$ There is exactly one value of (p, q).

88. **(b)** Given $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$

Given that $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}]=0$

i.e. $\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ x & (x-2) & -1\end{vmatrix}=0$

$\Rightarrow\ 1[1-2(x-2)]-1[-1-2x]+1[x-2+x]=0$

$\Rightarrow\ 1-2x+4+1+2x+2x-2=0$

$\Rightarrow\ 2x=-4 \Rightarrow\ x=-2$

89. **(b)** Given that $|2\hat{u}\times 3\hat{v}|=1$ and θ is acute angle between $\hat{u}$ and $\hat{v}$, $|\hat{u}|=1, |\hat{v}|=1$

$\Rightarrow\ |2\hat{u}\times 3\hat{v}|=6|\hat{u}||\hat{v}||\sin\theta|=1$

$\Rightarrow\ 6|\sin\theta|=1\ \Rightarrow\ \sin\theta=\frac{1}{6}$

Hence, there is exactly one value of θ for which $2\hat{u}\times 3\hat{v}$ is a unit vector.

90. **(d)** $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{a}.\vec{b}\neq 0$, $\vec{b}.\vec{c}\neq 0$

$\Rightarrow\ (\vec{a}.\vec{c}).\vec{b}-(\vec{b}.\vec{c})\vec{a}=(\vec{a}.\vec{c}).\vec{b}-(\vec{a}.\vec{b}).\vec{c}$

$\Rightarrow (\vec{a}.\vec{b}).\vec{c}=(\vec{b}.\vec{c})\vec{a}\ \Rightarrow\ \vec{a}\parallel\vec{c}$.

91. **(d)** Given that

$\vec{a}=\hat{i}-\hat{k},\ \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and

$\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}]=\vec{a}.(\vec{b}\times\vec{c})=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}$

$=1[1+x-y-x+x^2]-[x^2-y]$

$=1-y+x^2-x^2+y=1$

Hence $[\vec{a}\ \vec{b}\ \vec{c}]$ is independent of x and y both.

92. **(b)** Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

Given that

$\left[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}\right]=\left[\vec{a}\ \vec{b}+\vec{c}\ \vec{b}\right]$

$\Rightarrow\begin{vmatrix}\lambda(a_1+b_1) & \lambda(a_2+b_2) & \lambda(a_3+b_3)\\ \lambda^2 b_1 & \lambda^2 b_2 & \lambda^2 b_3\\ \lambda c_1 & \lambda c_2 & \lambda c_3\end{vmatrix}$

$=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$\Rightarrow\lambda^4\begin{vmatrix}a_1+b_1 & a_2+b_2 & a_3+b_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$

$=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$R_1\to R_1-R_2$ in 1st det.

and $R_2\to R_2-R_3$ in 2nd det.

$\Rightarrow\lambda^4\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=\begin{vmatrix}a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$\Rightarrow\lambda^4=-1$

Hence λ has no real values.

93. **(c)** Let $\vec{a}=x\vec{i}+y\vec{j}+z\vec{k}$

$\vec{a}\times\vec{i}=z\vec{j}-y\vec{k}\ \Rightarrow\left|\vec{a}\times\vec{i}\right|^2=y^2+z^2$

Similarly, $\left|\vec{a}\times\vec{j}\right|^2=x^2+z^2$ and $\left|\vec{a}\times\vec{k}\right|^2=x^2+y^2$

Adding all above equation

$\Rightarrow\ \left|\vec{a}\times\vec{i}\right|^2+\left|\vec{a}\times\vec{j}\right|^2+\left|\vec{a}\times\vec{k}\right|^2$

$=2(x^2+y^2+z^2)\ =2|\vec{a}|^2$

94. **(a)** Given that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Clearly $\vec{a}$ and $\vec{b}$ are non collinear

$\Rightarrow(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Comparing both side.

$\therefore\ \vec{a}.\vec{c}=0$ and $-\vec{b}.\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \cos\theta=\frac{-1}{3}$

$\therefore \sin\ \theta=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3}$

[θ is acute angle between $\vec{b}$ and $\vec{c}$]

95. **(c)** $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$

$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w})$

$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w})\quad [\because \vec{v}\times\vec{v}=0]$

$=\vec{u}.(\vec{u}\times\vec{v})-\vec{u}.(\vec{u}\times\vec{w})+\vec{u}.(\vec{v}\times\vec{w})+\vec{v}.(\vec{u}\times\vec{v})$

$-\vec{v}.(\vec{u}\times\vec{w})+\vec{v}.(\vec{v}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})+\vec{w}.(\vec{u}\times\vec{w})\ -\vec{w}.(\vec{v}\times\vec{w})$

We know that $[\vec{a},\vec{a},\vec{b}]=0$

$=\vec{u}.(\vec{v}\times\vec{w})-\vec{v}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})$

$=[\vec{u}\vec{v}\vec{w})]+[\vec{v}\vec{w}\vec{u})]-[\vec{w}\vec{u}\vec{v}]=\vec{u}.(\vec{v}\times\vec{w})$

96. **(b)** Normal vector of the face OAB

$$=\overrightarrow{OA}\times\overrightarrow{OB}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&1\\2&1&3\end{vmatrix}=5\hat{i}-\hat{j}-3\hat{k}$$

Normal vector of the face ABC

$$=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&2\\-2&-1&1\end{vmatrix}=\hat{i}-5\hat{j}-3\hat{k}$$

Angle between the faces = angle between their normals

$$\cos\theta=\left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right|=\frac{19}{35}\text{ or }\theta=\cos^{-1}\left(\frac{19}{35}\right)$$

97. **(a)** Given that $\vec{u}.\hat{n}=0$ and $\vec{v}.\vec{n}=0$

$\Rightarrow\ \vec{n}$ is perpendicular both $\vec{u}$ *and* $\vec{v}$,

$\therefore\ \hat{n}=\frac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|}$

$$\hat{n}=\frac{\begin{vmatrix}i&j&k\\1&1&0\\1&-1&0\end{vmatrix}}{\sqrt{2}\times\sqrt{2}}=\frac{-2\hat{k}}{2}=-k$$

$|\vec{\omega}.\hat{n}|=|(i+2j+3k).(-k)|=|-3|=3$

98. **(c)** Let $\vec{a}+\vec{b}+\vec{c}=\vec{r}$. Then $\vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{r}$

$\Rightarrow 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{a}\times\vec{r}$

$\Rightarrow \vec{a}\times\vec{b}-\vec{c}\times\vec{a}=\vec{a}\times\vec{r}\Rightarrow\vec{a}\times\vec{r}=\vec{0}$

$[\because\ \vec{a}\times\vec{b}=\vec{c}\times\vec{a}]$

Similarly $\vec{b}\times\vec{r}=\vec{0}\ \&\ \ \vec{c}\times\vec{r}=\vec{0}$

Above three conditions can be hold if and only if $\vec{r}=\vec{0}$

99. **(b)** We have $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-5&0\\6&3&0\end{vmatrix}=39\hat{k}=\vec{c}$

Also $|\vec{a}|=\sqrt{34},|\vec{b}|=\sqrt{45},|\vec{c}|=39$;

$\therefore\quad |\vec{a}|:|\vec{b}|:|\vec{c}|=\sqrt{34}:\sqrt{45}:39$.

100. **(a)** Given that $\vec{a},\vec{c},\vec{b}$ form a right handed system,

$$\therefore\quad \vec{c}=\vec{b}\times\vec{a}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&0\\x&y&z\end{vmatrix}=z\hat{i}-x\hat{k}$$

101. **(a)** $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$

$=(\vec{a}\times\vec{b}).\ \left\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\right\}$

$\because\ \vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

$=(\vec{a}\times\vec{b})\cdot\left\{(\vec{m}\cdot\vec{a})\vec{c}-(\vec{m}\cdot\vec{c})\vec{a}\right\}$ (where $\vec{m}=\vec{b}\times\vec{c}$)

$=\{(\vec{a}\times\vec{b})\cdot\vec{c}\}\cdot\{(\vec{a}\cdot(\vec{b}\times\vec{c})\}=[\vec{a}\ \vec{b}\ \vec{c}]^2=4^2=16$.

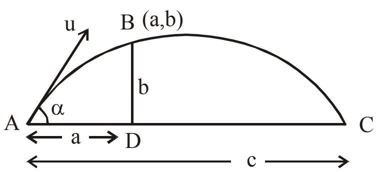
102. **(b)** Since, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\frac{\pi}{6}=4\times2\times\frac{\sqrt{3}}{2}=4\sqrt{3}$.

We know that, $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

$\Rightarrow\ (\vec{a}\times\vec{b})^2+48=16\times4$

$\Rightarrow\ (\vec{a}\times\vec{b})^2=16$

103. **(a)** Let B be the top of the wall whose coordinates will be (a, b). Range $(R)=c$



B lies on the trajectory

$\therefore\quad y=x\tan\alpha-\frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$

$\Rightarrow\ b=a\tan\alpha-\frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$

$\Rightarrow\ b=a\tan\alpha\left[1-\frac{ga}{2u^2\cos^2\alpha\tan\alpha}\right]$

$=a\tan\alpha\left[1-\frac{a}{\frac{2u^2}{g}\cos^2\alpha.\frac{\sin\alpha}{\cos\alpha}}\right]$

$=a\tan\alpha\left[1-\frac{a}{\frac{u^2.2\sin\alpha\cos\alpha}{g}}\right]$

$$= a\tan\alpha\left[1-\frac{a}{\frac{u^2\sin 2\alpha}{g}}\right]$$

$$= a\tan\alpha\left[1-\frac{a}{R}\right] \qquad \left(\because\ R=\frac{u^2\sin^2\alpha}{g}\right)$$

$$\Rightarrow\ b = a\tan\alpha\left[1-\frac{a}{c}\right]$$

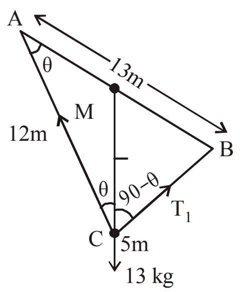
$$\Rightarrow\ b = a\tan\alpha\,.\left(\frac{c-a}{c}\right)$$

$$\Rightarrow\ \tan\alpha = \frac{bc}{a(c-a)}$$

The angle of projection,

$$\alpha = \tan^{-1}\frac{bc}{a(c-a)}$$

104. (a)



In ΔABC

$\because\ 13^2 = 5^2 + 12^2 \Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow \angle ACB = 90°$

M is mid point of the hypotenuse AB, therefore $MA = MB = MC$

$\Rightarrow \angle A = \angle ACM = \theta$

Applying Lami's theorem at C, we get

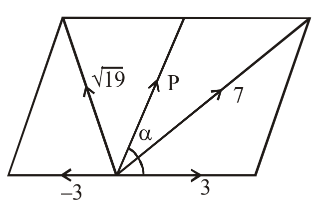
$$\frac{T_1}{\sin(180-\theta)} = \frac{T_2}{\sin(90+\theta)} = \frac{13kg}{\sin 90°}$$

$\Rightarrow\ T_1 = 13\sin\theta$ and $T_2 = 13\cos\theta$

$\Rightarrow\ T_1 = 13\times\frac{5}{13}$ and $T_2 = 13\times\frac{12}{13}$

$\Rightarrow\ T_1 = 5$ kg and $T_2 = 12$ kg

105. (c) Given that : Force $P = Pn$, $Q = 3n$, resultant $R = 7n$ & $P' = Pn$, $Q' = (-3)n$, $R' = \sqrt{19}\,n$



We know that $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$\Rightarrow\ (7)^2 = P^2 + (3)^2 + 2\times P\times 3\cos\alpha$

$\Rightarrow\ 49 = P^2 + 9 + 6P\cos\alpha$

$\Rightarrow\ 40 = P^2 + 6P\cos\alpha$ .....(i)

and $\left(\sqrt{19}\right)^2 = P^2 + (-3)^2 + 2P\times -3\cos\alpha$

$\Rightarrow\ 19 = P^2 + 9 - 6P\cos\alpha$

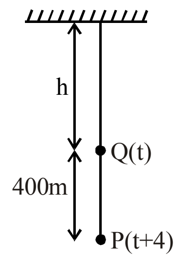
$\Rightarrow\ 10 = P^2 - 6P\cos\alpha$ .....(ii)

Adding (i) and (ii) $50 = 2P^2$

$\Rightarrow\ P^2 = 25 \Rightarrow P = 5n.$

106. (a) We know that $h = \frac{1}{2}gt^2$

and $h + 400 = \frac{1}{2}g(t+4)^2$



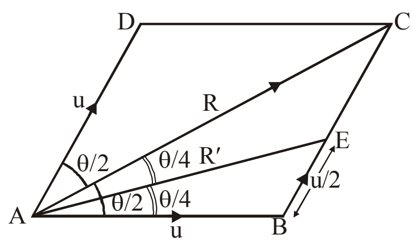
Subtracting, we get $400 = 8g + 4gt$

$\Rightarrow\ t = 8$ sec

$\therefore\ h = \frac{1}{2}\times 10\times 64 = 320m$

$\therefore$ Required height $= 320 + 400 = 720\ m$

107. (b) Let two velocities u and u at an angle θ to each other the resultant is given by



$R^2 = u^2 + u^2 + 2u^2\cos\theta = 2u^2(1+\cos\theta)$

$\Rightarrow R^2 = 4u^2\cos^2\theta/2$ or $R = 2u\cos\frac{\theta}{2}$

Now in second case, the new resultant AE (i.e., R') bisects $\angle CAB$, therefore using angle bisector theorem in ΔABC, we get

$$\frac{AB}{AC} = \frac{BE}{EC} \Rightarrow \frac{u}{R} = \frac{u/2}{u/2} \Rightarrow R = u$$

$$\Rightarrow 2u\cos\frac{\theta}{2} = u$$

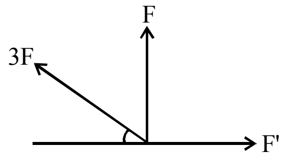
$$\Rightarrow\ \cos\frac{\theta}{2} = \frac{1}{2} = \cos 60° \Rightarrow \frac{\theta}{2} = 60°$$

or $\theta = 120°$

108. (d) According to question $F' = 3F\cos\theta$ and $F = 3F\sin\theta$

$\Rightarrow F' = 2\sqrt{2}\,F$

$\Rightarrow F : F' :: 3 : 2\sqrt{2}$.



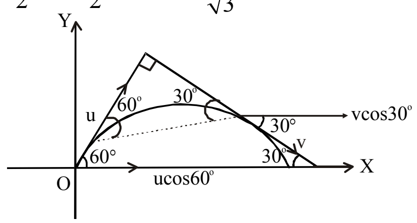
109. (b) Let A and B be displaced by a distance x then Change in moment of $(A+B)$ = applied moments

$$\Rightarrow (A+B)\times x = H \Rightarrow x = \frac{H}{A+B}$$

110. (d) As per question $u\cos 60° = v\cos 30°$

(as horizontal component of velocity remains the same)

$$\Rightarrow u\cdot\frac{1}{2} = v\cdot\frac{\sqrt{3}}{2} \quad \text{or} \quad v = \frac{1}{\sqrt{3}}u$$



111. (b) For same horizontal range the angles of projection must be α and $\frac{\pi}{2}-\alpha$

$$\therefore \quad t_1 = \frac{2u\sin\alpha}{g} \quad \text{....(i)}$$

$$t_2 = \frac{2u\sin\left(\frac{\pi}{2}-\alpha\right)}{g} = \frac{2u\cos\alpha}{g} \quad \text{....(ii)}$$

Squaring and adding eqn. (i) and (ii),

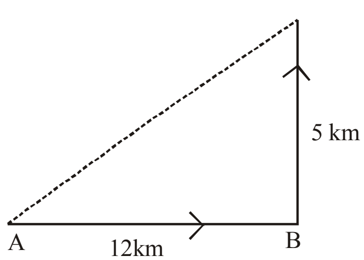
$$\therefore \quad t_1^2 + t_2^2 = \frac{4u^2}{g^2}$$

112. (a) Given $v = \frac{1}{4}$ m/s, component along OB

$$= \frac{v\sin 30°}{\sin(45°+30°)} = \frac{\frac{1}{4}\times\frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{6}-\sqrt{2}}{8}$$

113. (d) Time taken by the particle in complete journey

$$T = \frac{12}{4}+\frac{5}{5} = 4\ hr.$$



$$\therefore \text{ Average speed } = \frac{12+5}{4} = \frac{17}{4}$$

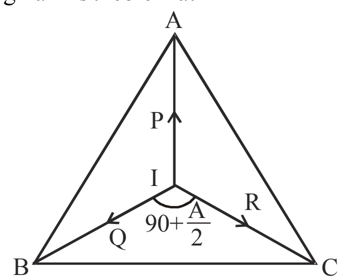
$$\text{Average velocity } = \sqrt{\frac{12^2+5^2}{4}} = \frac{13}{4}$$

114. (d) Let I is incentre of ΔABC.

$\therefore$ IA, IB, IC are bisectors of the angles A, B and C.

Now $\angle BIC = 180 - \frac{B}{2} - \frac{C}{2} = 90° + \frac{A}{2}$ *etc.*

Applying Lami's theorem at I

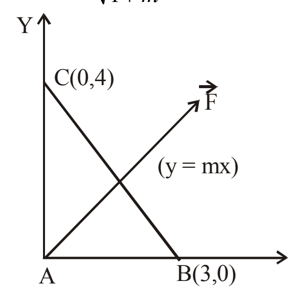


$$\frac{P}{\sin(90°+\frac{A}{2})} = \frac{Q}{\sin\left(90°+\frac{B}{2}\right)} = \frac{R}{\sin\left(90°+\frac{C}{2}\right)}$$

$$\Rightarrow \quad P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$$

115. (c) Since, the moment about A is zero, hence $\vec{F}$ passes through A. Taking A as origin. Let the line of action of force $\vec{F}$ be $y = mx$. (see figure)

$$\text{Moment about } B = \frac{3m}{\sqrt{1+m^2}}|\vec{F}| = 9 \quad \text{....(1)}$$



$$\text{Moment about } C = \frac{4}{\sqrt{1+m^2}}|\vec{F}| = 16 \text{....(2)}$$

Dividing (1) by (2), we get

$$m = \frac{3}{4} \Rightarrow |\vec{F}| = 5N.$$

116. (c) Let forces be P and Q. then $P + Q = 4$ ....(1)

and $P^2 + Q^2 = 3^2$ ....(2)

Solving eqns. (1) and (2), we get the forces

$$\left(2+\frac{\sqrt{2}}{2}\right)N \text{ and } \left(2-\frac{\sqrt{2}}{2}\right)N$$

117. (d) Resultant of forces

$\vec{F} = 4\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} + \hat{j} - \hat{k} = 7\hat{i} + 2\hat{j} - 4\hat{k}$

Displacement

$\vec{d} = 5\hat{i} + 4\hat{j} + \hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k}) = 4\hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore$ Work done $= \vec{F}.\vec{d} = 28 + 4 + 8 = 40$

118. (a) Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

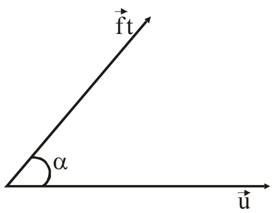
We have $R_1 = \dfrac{u^2}{g(1+\sin\beta)}$ and $R_2 = \dfrac{u^2}{g(1-\sin\beta)}$

Adding above equations

$\dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{2g}{u^2}$ or $\dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{2}{R}$ $\left[\because R = \dfrac{u^2}{g}\right]$

$\therefore R_1, R, R_2$ are in H.P.

119. (a) Let the two velocities be $\vec{v}_1 = u\hat{i}$ and $\vec{v}_2 = (ft\cos\alpha)\hat{i} + (ft\sin\alpha)\hat{j}$



$\therefore$ Relative velocity of second with respect to first

$\vec{v} = \vec{v}_2 - \vec{v}_1 = (ft\cos\alpha - u)\hat{i} + ft\sin\alpha\,\hat{j}$

$\Rightarrow |\vec{v}|^2 = (ft\cos\alpha - u)^2 + (ft\sin\alpha)^2$

$= f^2t^2 + u^2 - 2uft\cos\alpha$

For $|\vec{v}|$ to be min and max. we should have

$\dfrac{d|v|^2}{dt} = 0 \Rightarrow 2f^2t - 2uf\cos\alpha = 0$

$\Rightarrow t = \dfrac{u\cos\alpha}{f}$

Also $\dfrac{d^2|v|^2}{dt^2} = 2f^2 = +ve$

$\therefore |v|^2$ and hence $|v|$ is least at the time $\dfrac{u\cos\alpha}{f}$

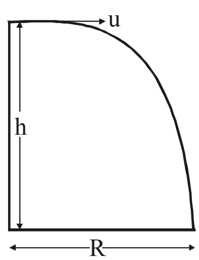
120. (a) Given that the stone projected horizontally. For horizontal motion,

Distance = speed × time $\Rightarrow R = ut$

and for vertical motion

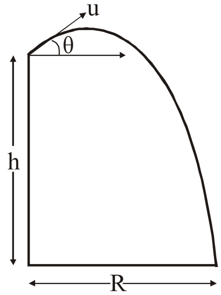
$h = 0 \times t + \dfrac{1}{2}gt^2$

$\Rightarrow t = \sqrt{\dfrac{2h}{g}}$



$\therefore$ We get $R = u\sqrt{\dfrac{2h}{g}}$ ...(1)

When the stone projected at an angle θ, for horizontal and vertical motions, we have



$R = u\cos\theta \times t$ ....(2)

and $h = -u\sin\theta \times t + \dfrac{1}{2}gt^2$ ....(3)

From eqns. (1) and (2) we get

$u\sqrt{\dfrac{2h}{g}} = u\cos\theta \times t$

$\Rightarrow t = \dfrac{1}{\cos\theta}\sqrt{\dfrac{2h}{g}}$

Putting the value of t in eq (3) we get

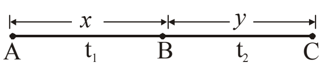
$h = -\dfrac{u\sin\theta}{\cos\theta}\sqrt{\dfrac{2h}{g}} + \dfrac{1}{2}g\left[\dfrac{2h}{g\cos^2\theta}\right]$

$h = -u\sqrt{\dfrac{2h}{g}}\tan\theta + h\sec^2\theta$

$h = -u\sqrt{\dfrac{2h}{g}}\tan\theta + h\tan^2\theta + h$

$\tan^2\theta - u\sqrt{\dfrac{2}{hg}}\tan\theta = 0; \therefore \tan\theta = u\sqrt{\dfrac{2}{hg}}$

121. (a) Let the body travels from A to B with constant acceleration t and from B to C with constant retardation r.



If $AB = x$, $BC = y$, time taken from A to $B = t_1$ and time taken from B to $C = t_2$, then $s = x + y$ and $t = t_1 + t_2$

For the motion from A to B

$v^2 = u^2 + 2fs \Rightarrow v^2 = 2fx\ (\because u = 0)$

$\Rightarrow x = \dfrac{v^2}{2f}$ ....(1)

and $v = u + ft \Rightarrow v = ft_1$

$\Rightarrow t_1 = \dfrac{v}{f}$ ...(2)

For the motion from B to C

$v^2 = u^2 + 2fs$

$\Rightarrow \quad 0 = v^2 - 2ry \quad \Rightarrow y = \dfrac{v^2}{2r}$ ...(3)

and $\quad v = u + ft \;\Rightarrow 0 = v - rt_2$

$\Rightarrow \quad t_2 = \dfrac{v}{r}$

Adding equations (1) and (3), we get

$$x + y = \frac{v^2}{2}\left[\frac{1}{f} + \frac{1}{r}\right] = s$$

Adding equations (2) and (4), we get

$$t_1 + t_2 = v\left[\frac{1}{f} + \frac{1}{r}\right] = t$$

$$\therefore \quad \frac{t^2}{2s} = \frac{v^2\left[\frac{1}{f} + \frac{1}{r}\right]^2}{2 \times \frac{v^2}{2}\left(\frac{1}{f} + \frac{1}{r}\right)} = \frac{1}{f} + \frac{1}{r}$$

$$\Rightarrow \quad t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$$

122. **(c)** $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ .... (1)

When $\vec{Q}$ and $\vec{R}$ are doubled

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ .... (2)

When $\vec{Q}$ is reversed and $\vec{R}$ is doubled

$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ .... (3)

Adding (1) and (3), $5R^2 = 2P^2 + 2Q^2$

$\Rightarrow 2P^2 + 2Q^2 - 5R^2 = 0$ .... (4)

Applying $(3) \times 2 + (2)$, $12R^2 = 3P^2 + 6Q^2$

$\Rightarrow 3P^2 + 6Q^2 - 12R^2 = 0$ ....(5)

From (4) and (5)

$$\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$$

$$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6} \; or \; P^2 : Q^2 : R^2 = 2:3:2$$

123. **(c)** We know that $\vec{G} = \vec{r} \times \vec{p}$; $|\vec{G}| = |\vec{r}||\vec{p}|\sin\theta$

$|\vec{H}| = |\vec{r}||\vec{p}|\cos\theta \qquad [\because \sin(90^o + \theta) = \cos\theta]$

$G = |\vec{r}||\vec{p}|\sin\theta$ .....(1)

$H = |\vec{r}||\vec{p}|\cos\theta$ .....(2)

$x = |\vec{r}||\vec{p}|\sin(\theta + \alpha)$ .....(3)

From (1), (2) & (3), $\quad x = \vec{G}\cos\alpha + \vec{H}\sin\alpha$.

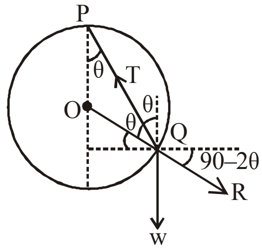
124. **(d)** $\vec{F} = \vec{F}_1 + \vec{F}_2 = 7i + 2j - 4k$

$\vec{d}$ = Positon Vector of $\vec{B}$ – Position Vector of $\vec{A}$

$= 4i + 2j - 2k$

$W = \vec{F}.\vec{d} = 28 + 4 + 8 = 40$ unit

125. **(b)** From figure $\angle TQW = 180 - \theta$; $\angle RQW = 2\theta$; $\angle RQT = 180 - \theta$



Applying Lami's theorem at Q.

$$\frac{T}{\sin 2\theta} = \frac{R}{\sin(180-\theta)} = \frac{W}{\sin(180-\theta)}$$

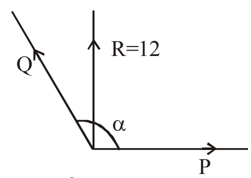
$\Rightarrow \quad R = W$ and $T = 2W\cos\theta$

126. **(a)** Given that $P + Q = 18$ ....(1)

We know that

$P^2 + Q^2 + 2PQ\cos\alpha = 144$ ......(2)

$$\tan 90^o = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$



$\Rightarrow \quad P + Q\cos\alpha = 0$ ......(3)

From (2) and (3),

$Q^2 - P^2 = 144 \Rightarrow (Q-P)(Q+P) = 144$

$\therefore \quad Q - P = \dfrac{144}{18} = 8$

From (1), On solving, we get $Q = 13, P = 5$